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THE
DOCTRINE
OF
PROJECTILES

Demonstrated and Apply'd to all the
most useful PROBLEMS in practical
GUNNERY.

To which is Added,

The Description and Use of a new
Mathematical INSTRUMENT.

Together with several curious Pro-
perties of PROJECTILES never
before Publish'd.

By *WILLIAM STARRAT.*

D U B L I N:

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near Essex-street. M.DCC.XXXIII.

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LONDON:
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To the LEARNED the
PROVOST, FELLOWS,
and SCHOLARS,
OF
TRINITY COLLEGE,

The following
DOCTRINE
OF
PROJECTILES, &c.

IS
Humbly Dedicated

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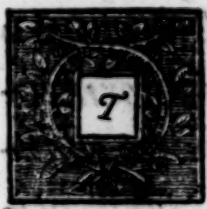
To the Author

THE PREFACE



T H E

P R E F A C E.



THE general Part of what I here offer to the Publick was first written for the Use of my Scholars; there being no Book extant that render'd the Subject sufficiently intelligible to such as had made but a small Progress in Mathematicks.

Though I am far from thinking my self the fittest Person to undertake a Task

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Task of this Kind, I am of Opinion such as could perform it better, will not be the first in finding Fault, and am confident my weak Endeavours will be very acceptable to low Proficients who are real Lovers of this Kind of Learning.

I also flatter my self with Hopes of the kind Sentiments of the learned and best Judges of the Subject: As well because I have given several Things intirely new, and resolv'd several Problems never before insisted on, as that I have rendered the Solutions of most of the common Cases in practical Gunnery much shorter and easier than formerly; having made several Things obvious by Inspection, whose Demonstrations, by the best Geometer that ever wrote for the Use of practical Engineers, were drawn from an affected Quadratick Equation, rais'd from Properties of the Parabola suppos'd to be known, or from pretty difficult

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ficult Lemmata. And though he has reduc'd to a few Pages what before made no small Volume, I am persuaded he would not only excuse but approve any ones Labour, though it occupied a little more Paper, who had deduced those Rules, and demonstrated all the Laws of Motion and Properties of the Parabola requisite to the Knowledge of them, from no higher Principles than simple Equations, and those of plain Trigonometry.

Let none think I am here preferring, or any Way comparing my self to the learned Dr. Halley. I am so far from having that Vanity, that I am persuaded, if he had taken half the Time or Pains I have taken to level that Subject to lower Capacities, his wou'd have render'd the Endeavours of much abler Pens than mine vain and useless.

And here to obviate some Objections that may be made, particularly, that I have made no Allowance for the Resistance

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sistance of the Air, and suppose the Lines of descent of heavy Bodies to be parallel to each other, I shall give the Opinion of that celebrated Author who says,

'Tis certain that in large Skot of Metal, whose Weight many thousand Times surpasses that of the Air, and whose Force is very great in Proportion to the Surface wherewith they press thereon, this Opposition, or Resistance of the Air, is scarcely discernible.

And since the Arch of our Earth over which any of our Engines can cast a Project, is so very small, we may well enough suppose the Descent of heavy Bodies to be in parallel Lines, the Difference being so small as by no Means to be discover'd in Practice.

And the ingenious Dr. Keill who wrote lately of Projectiles in his small but excellent Treatise of natural Philosophy

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sophy makes no Computation or Allowance for either.

'Tis true the immortal Sir Isaac Newton shews how those Things may be computed, and what the projectile Curve differs from a true Parabola, in Mediums whose Resistance bears any considerable Proportion to the Weight and Force of the Bodies that are projected in them. But whoever undertakes to reduce his Doctrine to practical Use in Gunnery, will find it a very hard Task.

I have now some Sheets by me, wherein I have gone as far that Way, as perhaps any, that will make the Objection, ever did: But being discourag'd by the tediousness of the Demonstrations, and the perplexedness of the Calculations, I thought proper to leave it out, and in its Room to give the Description and Use of an Instrument, which will be of much greater Service to practical Engineers.

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Some Readers may accuse me of being too prolix, having made several Things Theorems which to a ready Genius were sufficiently evident as Corollaries to foregoing Ones; as also having resolv'd Cases which seldom occur in Practice. But so difficult a Task it is to please all Tastes, and suit all Capacities, that I'm persuaded there are many who will alledge I have not made some Things sufficiently plain, and that several Cases may occur which I have not mention'd. The judicious and experienc'd Reader who knows the Difficulty and almost Impossibility of avoiding both these Extreams, will, I hope, absolve me from those Charges of the querulous on either Side: And indeed I must say, however presumptuous upon first hearing it may seem, that with greatest Confidence I submit my self and Performance with all its Blemishes, to the judicious Reader; for as he will be the first to discern the Defects either in the
Matter

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Matter or Diction, so will be ever be the readiest to make Allowances in Favour of an Author, whose sole Ambition was to become in some Measure useful to the Publick: In this Assurance easy, if Objections are made thro' Ignorance or Want of Experience, I can excuse them; if through Malice, I despise them.



E R R A T A.

PAGE 52, Line 11, for *felling*, read *falling*. p. 83, l. 16, for *AM*, r. *Am*. p. 101, l. 3, for *AX*, r. *Ax*. p. 132, l. 12, for *AX*, r. *AN*. p. 137, l. 9, for *Am*, r. *AM*. p. 138, l. 15, for *A to Q*, r. *A to K*. p. 167, l. 2, for *2d*, r. *3d*. l. 6, for *half the Sum*, r. *the Sum*. l. 9, for *half the Difference*, r. *the Difference*. p. 152, l. 7, 10, for *HO*, r. *AO*.

Case 50th, the Angles *D* and *M* ought to be $45^{\circ} 44'$, and $121^{\circ} 16'$, which will alter all the Figures in the rest of that Case, tho' the Statings by the Letters are right.

Figure 9th. in the Plates, put *t* to the End of the Line *TH* produc'd.

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The DOCTRINE of
PROJECTILES,
Demonstrated from the LAWS of
MOTION
AND
GRAVITATION.

CHAP. I.
Of SIMPLE MOTION.

DEFINITIONS.

I. **M**OTION, is a continual
and successive Change of
Place.

B

II.

2 *Of Simple Motion.*

II. *Celerity*, is an Affection of Motion, whereby a Body in Motion, passes over a given Space, in a given Time.

III. *Equable Motion*, is that whereby a moving Body, passes over equal Spaces in equal Times.

IV. *Motion equally accelerated*, is that to which, in equal Times, there is always added equal Increments of Velocity.

V. *Motion equably retarded*, is that whose Velocity in equal Times equally decreases.

VI. A *Momentum*, is that Power, or Force, incident to moving Bodies, whereby they continually tend from their present Places.

AXIOMS.

Of Simple Motion. 3

A X I O M S.

I. No single Force, or Power, can act, at the same Instant of Time, with two different Directions; nor, consequently, give Motion to a Body in another Direction than that in which it is apply'd to the Body; hence all Simple Motion is rectilineal.

II. If any Power, apply'd to a Body, gives it a certain Degree of Motion, double that Power, apply'd, will give double that Motion, and triple, triple that Motion, &c. for Effects are proportional to their Causes.

III. If a moving Body, is directly oppos'd, by any Power, which is to the Momentum, or Force wherewith the Body moves, in any given Ratio; that Power, will destroy a proportional Part of the Momentum of the Bo-

B 2

dy;

4 *Of Simple Motion.*

dy; and if the moving Body, is oppos'd, by any Number of Powers successively, the aggregate of which, being equal to the Momentum of the Body, its Motion will be quite destroy'd.

IV. Hence, if a Body, moving with any given finite Force, however great, be constantly oppos'd, by any given Power however small, the Motion will be, at last destroy'd; and the Power, continuing to act in its first direction, will give the Body a Motion the contrary Way.

L A W I.

If a Body, be put in Motion, in free Space, where no extraneous Cause affects it, it will continue to move, uniformly in a right Line, according to the Direction and Celerity first impressed upon it.

If

Of Simple Motion. 5

If the Body continues to move by Means of the first Impulse, the Motion must be rectilinear, (*Ax. 1.*) and, since all other Obstacles are supposed to be removed, it must continue to move, unless it has a Power to destroy its own Motion; but since a Body may be put in Motion according to any Direction, if it has a Power to destroy it's own Motion, that Power must be such, as can act according to any Direction, and cease to act when the Occasion is over; otherwise it wou'd give the Body a Motion the contrary Way, (*Ax. 4.*) which Power is absurd to be supposed in any inanimate Body. This is Part of the great Sir *Isaac Newton's* first Law of Nature, and is the Foundation of all Laws of Motion.

L A W

6 Of Simple Motion.

L A W II.

The Change of Motion is always proportionable to the moving Force impressed, and is always made according to the Right-Line in which that Force is impressed.

The first Part of this Law is plain from the second *Axiom*, for if any Force generates a certain Motion or Change of Motion, double that Force will generate double, and triple, triple, &c. and because this Motion, or Change in Motion, is always determin'd by the moving Force, it will be made according to its Direction; (by *Law* 1.) for if the Body was in Motion before, and if the new impressed Force was in the same Direction, it will add to the former Motion, if opposite to it, it will deduct from it; if impressed in an oblique

Of Simple Motion. 7

lique Direction, the Change will be obliquely ; but still the Acceleration, Retardation or Obliquity, will be in Proportion to the new impressed Force. This Law will be more strictly demonstrated in *Theorem* 1st. of compound Motion.

SCHOLIUM I.

Since a Body can neither give nor destroy its own Motion, nor alter its Celerity, the Momentum, or Force wherewith it moves, must still be equal to the moving Force impressed ; and since Extension or Figure, can no way affect Motion in pure Space, there is nothing in Bodies, that can require different Forces, to move them with equal Celerities, but their different Quantities of Matter.

Let

8 *Of Simple Motion.*

Let B & b , express the Quantities of Matter, in two Bodies, C & c , their respective Celerities.

M & m , their Momenta, or Forces with which they move.

L A W III.

If $C = c$. $M : m :: B : b$, that is, If the Celerities are equal, the Momenta will be as the Quantities of Matter in the moving Bodies.

It's plain, that whatever Proportion B has to b , the Force that move them equally, must have the same Proportion; (*per Scholium*) but the Momenta and moving Forces are equal, and therefore have the same Proportion to the Quantities of Matter in the moving Bodies.

L A W

L A W IV.

If $B = b$. $M : m :: C : c$. that is,
If the Quantities of Matter in the
moving Bodies are equal, the Mo-
menta will be as their respective Ce-
lerities.

If the Bodies are equal, the Cele-
rities must be as their respective
Causes, which are the moving Forces :
but the Momenta and moving Forces
are equal, (by *Schol. 1.*) and there-
fore in the same Ratio to the Cele-
rities.

L A W V.

Universally, $M : m :: B \times C : b \times c$.
that is, in all Cases, the Ratio of
the Momenta is composed of the
Ratio's of the Quantities of Matter
C in

10 *Of Simple Motion.*

in the moving Bodies, and their Celerities conjointly.

Let $D (= B)$ be moved with c , the Celerity of b , and let F , be it's Momentum; then (by *Law 3d.*) because $c = c$. $D : b :: F : m$. whence $D \times m = b \times F$, and (by *Law 4th.*) because $D = B$. $C : c :: M : F$. whence $C \times F = c \times M$. and putting B in the Place of D , in the first Equation, and multiplying it by the last, you have $B \times C \times m \times F = b \times c \times M \times F$, and dividing both Sides by F , the Equation is $B \times C \times m = b \times c \times M$. which gives this Analogy $M : m :: B \times C : b \times c$. Q. E. D.

L A W VI.

$C : c :: b \times M : B \times m$. that is, The Ratio of the Celerities is composed of the direct Ratio of the Momenta, and the inverse Ratio of the Quantities of Matter. By

Of Simple Motion. II

By the last, $B \times C \times m = b \times c \times M$.
which gives this Analogy, $C : c ::$
 $b \times M : B \times m.$ Q. E. D.

Whence *LAW VII.*

If $M = m$, $C : c :: b : B$. that is,
If the Momenta are equal, the Cele-
rities will be inverfely as the Quan-
tities of Matter in the moving Bo-
dies.

In comparing Motions, with re-
spect to the Length of Spaces passed
over by moving Bodies : Let C & c
represent their Celerities, as before,
 S & s , the Spaces passed over in T & t ,
the Times in which they move.

LAW VIII.

If $T = t$, $S : s :: C : c$. that is, If
the Times are equal, wherein Bodies
move, the Spaces passed over, will
C 2 be

12 *Of Simple Motion.*

be as the Celerities, with which they move.

L A W IX.

If $C = c$. $S : s :: T : t$, that is, If the Celerities are equal, the Spaces passed over by the moving Bodies, will be directly as the Times in which the Motions are made.

The Truth of these two *Laws* is obvious to any one who can conceive, that if a Body moves with double, triple, &c. the Celerity of another Body, it will, in an equal Time, pass over double, triple, &c. the Space which that other Body passes over; and if two Bodies move equally swift, if one continue moving double, or triple, &c. the Time the other moves, it will pass over double, or triple, &c. the Space which the other passes over.

L A W

L A W X.

Universally, $S : s :: T \times C : t \times c$.
that is, In all cases, the Ratio of the
Spaces passed over is compounded of
the Ratio's of the Times and Celerities
conjunctly.

Let D , be a Space passed over in
the Time T , wherein S was passed
over, and let it be passed over with
the Celerity c , wherewith s was
passed over; then (by *Law 9th.*)
 $T : t :: D : s$. and (by *Law 8th.*)
 $c : C :: D : S$. from the first Analogy
you have this Equation, $T \times s =$
 $t \times D$, and from the last Analogy,
 $c \times S = C \times D$, which two Equations
multiply'd, gives $T \times s \times C \times D = t \times D$
 $\times c \times S$; and dividing by D , you have
 $T \times s \times C = t \times c \times S$, which turn'd into
an Analogy, gives $S : s :: T \times C : t \times c$.
Q. E. D.

L A W

14 *Of Simple Motion.*

L A W XI.

If $S=s$, $C:c::t:T$. that is, If the Spaces passed over are equal, the Celerities are in the reciprocal Ratio of the Times.

By the last, $S:s::T\times C:t\times c$, and if $S=s$; $T\times C=t\times c$, which gives this Analogy, $C:c::t:T$. *Q. E. D.*

L A W XII.

$T:t::S\times c:s\times C$. that is, The Ratio of the Times is compounded of the direct Ratio of the Spaces passed over, and the reciprocal Ratio of the Celerities.

By *Law 10th*, $S:s::T\times C:t\times c$, which gives this Equation, $T\times C\times s=t\times c\times S$, which turn'd to an Analogy, gives $T:t::S\times c:s\times C$. *Q. E. D.*
L A W

L A W XIII.

$C:c::t \times S:T \times s$, that is, The Ratio of the Celerities is compounded of the direct Ratio of the Spaces passed over, and the reciprocal Ratio of the Times. For, by the last Equation, $T \times C \times s = t \times c \times S$, which gives $C:c::t \times S:T \times s$. *Q.E.D.*

L A W XIV.

$T:t::\frac{S}{C}:\frac{s}{c}$, that is, The Ratio of the Times is compounded of the direct Ratio of the Spaces passed over, apply'd to, or divided by the Celerities.

By the 10th. *Law*, $T \times C:t \times c::S:s$, and dividing the 1st. and 3d. by C , and the 2d. and 4th. by c , you have

$$T:t::\frac{S}{C}:\frac{s}{c}.$$

Q.E.D.

SCHO-

16 *Of Simple Motion.*

SCHOLIUM II.

If any determined Length of Space and Time are made the Integers ; as an Inch, a Foot, or a Yard, &c. a Second, a Minute, an Hour, &c. from any two of S , T , & C given, the third may be found ; for, $S = T \times C$, and $C = \frac{S}{T}$, also $T = \frac{S}{C}$. *E. g.* If C the Celerity, be at the Rate of 6 Feet in a Second of Time, and if T be 60 Seconds, or 1 Minute, then will $T \times C = 60 \times 6 = 360 = S$ the Space passed over in a Minute, &c. as before.

In like Manner, if B consists of 8 such Parts, as b contains 6, and if C , be to c , as 7 : 5, then will M , the Momentum of B , be to m , the Momentum of b , as 56, to 30, for, by the 5th. *Law*, $M : m :: B \times C : b \times c$,
that

Of Simple Motion. 17

that is, as 8×7 : to 6×5 . But M , B & C , may be determined by common Weight and Measure, for, by the best Experiments yet known, the Perpendicular Descent of a heavy Body in Vacuum is 16.14 Feet in a Second of Time, in which Fall it will acquire a Celerity double that Space, or 32.28, in a Second, as shall be afterward shewn, and, by *Page 54* of *Dr. Grausand's* Experiments, if the falling Body B , is a Pound-Weight, it's Momentum at the End of the Fall will be 42 Pounds nearly, and, by *Law 4th*, $\frac{M \times c}{C} = m$,

whence if c be equal to one Foot in a Second of Time, $\frac{42}{32.28} = 1.3 =$

m the Momentum or Force of a Body of one Pound-Weight, moving at the Rate of one Foot in a Second. Hence,

D

And,

18 *Of Simple Motion.*

And, from *Law* 5th, let B be any Number of Pounds, and C any Number of Feet, which B moves in a Second of Time, $B \times C \times 1.3$ will still be equal to M the Momentum.

E X A M P L E.

If a Cannon-Ball of 60 Pounds-Weight move at the Rate of 1000 Feet in a Second, it will have a Momentum, or Force, equal to $60 \times 1000 \times 1.3 = 78$ thousand Pounds-Weight.

C H A P. II.

Of Compound Motion and Oblique Percussion.

T H E O R E M I.

IF a Body at *A* (*Fig. 1.*) is impelled, at the same Time, by two Forces, which are to each other as the Right-Lines *AB*, *AC*, and in the same Direction with those Lines, it will pass through the Diagonal of the Parallelogram *AD*, in the same Time that it wou'd pass through either of the Sides *AB*, or *AC*, if only one of those Forces acted.

20 *Of Compound Motion.*

DEMONSTRATION.

Suppose the Body to move uniformly along the Right-Line AC , in the same Time that the Right-Line AC , keeping its Parallels Position, is mov'd uniformly along AB , when the Body wou'd have pass'd over Aa , in the Right-Line AC , AC will have pass'd over AG , a proportional Part of AB . make $Gg = Aa$, and the Body will be at g ; but because $AB : AC :: AG : Aa$, that is, $AB : BD :: AG : Gg$, the Triangles ABD & AGg are similar (by 5. *El*: 6.) and consequently the Point g , where the Body is, is in the Diagonal AD . Now, since AC , or GG , in its Passage along AB , is still parallel to BD , when AC coincides with BD , the Body will be at D , and in the whole Passage from A to D , it will still be $AB : BD :: AG : Gg$. a Body therefore impelled by two Forces,
which

Of Compound Motion. 21

which are to each other as the Sides of a Parallelogram, will move through the Diagonal of that Parallelogram, in the same Time that it wou'd move through either of the Sides, if only one of the Forces impelled it. *Q. E. D.*

C O R. I.

Hence, the joint Forces *AB, AC*, in that Position, is to either of the single Forces, as the Diagonal *AD*, to the respective Side which represents that Force.

For, since the Body is the same, the Forces, or Momenta, are as the Celerities, (by *Law* 4th.) and since the Times are equal, the Celerities are as the Spaces passed over, (by *Law* 8.) the Forces are therefore as the Spaces passed over ; that is, as the Diagonal to each respective Side.

C O R.

22 *Of Compound Motion.*

C O R. II.

Hence, if a Force which is as AD , acts with a contrary Direction DA , to the other two Forces AB , AC , the Body acted upon will be in Equilibrio.

C O R. III.

Hence, if a Body is kept in Equilibrio by three Powers, those Powers will be to each other, as the Sides of a Triangle form'd by Right-Lines drawn parallel to the respective Directions of those Powers.

C O R. IV.

Hence, any Force may be resolv'd into, and consider'd as, two, or more Forces.

THE O-

THEOREM II.

If a Body strikes a firm Obstacle CD (*Fig. 2d.*) obliquely, in the Direction AD , the Magnitude of the oblique Stroke will be to the Magnitude of the Stroke, which the same Body wou'd have produc'd if it had struck the Obstacle perpendicularly, or in the Direction AC , as the Sine of the Angle of Incidence ADC , to the Radius.

DEMONSTRATION.

Draw AB parallel to CD , and BD parallel to AC , then the whole Force wherewith the Body moves in the Right-Line AD , may be resolv'd into the two Forces AB , AC ; but the Force AB can no Way impinge on the Obstacle CD , because it is parallel to its Surface; the Force therefore
where-

24 *Of Compound Motion.*

wherewith the Body impinges on CD , is only AC ; but AC , is to AD the whole Force, as the Sine of the Angle of Incidence ADC , to the Radius.

Q. E. D.

C O R.

Since every oblique Stroke, is to the direct Stroke, as the Sine of the Angle of Incidence, to the Radius; they are to each other, as the Sines of their respective Angles of Incidence.

THEOREM III.

If a Body is impelled by any Force in the Direction AB , (*Fig. 3d.*) and if in its Motion towards B it be successively impelled by any Number of Forces, whose Directions AC , EG , HK , &c. are all parallels to BN , the Body will pass over the Spaces AE , EH , HN , and meet with the Right-Line

Of Compound Motion. 25

Line BN , in the same Time that it wou'd arrive at the Point B , if only the first Force had impelled it in the Direction AB .

DEMONSTRATION.

Since, (by *Theor.* 1.) the Diagonal AD wou'd be pass'd over by the joint Forces which are as AB , and AC , in the same Time that AB wou'd be pass'd over by the single Force AB ; let the Force EG meeting with the Body at the Point E , be to the Force wherewith the Body moves in the Diagonal AD , as EG , to ED , and the Diagonal EF will be pass'd over in the same Time that ED wou'd; let also HK be to the Force wherewith the Body moves in the Diagonal EF , as HK , to HF , and HN will be passed over in the same Time that HF wou'd, since AB wou'd be pass'd over in the same Time that AD wou'd, and AD ,
E in

26 Of Compound Motion:

in the same Time that $AE + EF$ wou'd, and $AE + EF$ in the same Time that $AE + EH + HN$, wou'd, consequently, $AE + EH + HN$ wou'd be pass'd over by the joint Forces AB , AC , EG , HK , in the same Time that AB wou'd, by the single Force AB .
Q. E. D.

C O R. I.

Hence, the Action of any Number of Forces whatever, apply'd to a moving Body in parallel Directions, alter not the Progress of that Body with respect to the parallel Distances. For, the Points e , & E , b , & H , B , & N , wou'd be pass'd through in equal Times in both Cases; nor does the Action of the Power AB alter the Progress of the Body, with respect to the Distances parallel to it: For, AC , EG , HK , have the same Effects, (*viz.*) BD , DF , FN , that they wou'd have had

Of Compound Motion. 27

had in the same Time with respect to the Motion towards the Line LN , parallel to AB , as if the Force AB had not acted at all.

C O R. II.

Hence, if the Forces AC, EG, HK , &c. are infinite in Number, and placed at infinitely small Distances, or, which is the same, if any one Force as that of Gravity, acted continually on the moving Body, its Way, or Path A, E, H, N , wou'd be a Curve.

C H A P. III.

Of the Descent of Heavy Bodies.

THEOREM IV.

THE Motion of a heavy Body falling directly downwards by its own Gravity, in free Space, is a Motion equally accelerated.

DEMONSTRATION.

Let the Time in which the Body is falling be suppos'd to be divided into very small equal Particles, and let Gravity acting the first Particle of Time draw the Body towards the Center, if
after

of Heavy Bodies. 29

after that first Particle of Time, Gravity wou'd cease to act the Motion receiv'd from it wou'd continue, and the Body wou'd move equally towards the Center, (by *Law 1.*) but as the Body is still heavy, and Gravity acts without Intermission, it will, in the 2d. Particle of Time add another Impulse to the Body equal to the former, and the Velocity of the Body will then be double to the former Velocity; and if the Force of Gravity was then quite remov'd, the Body wou'd continue to move in the same Right-Line with the Velocity then acquired; but in the 3d. Particle of Time the Body is acted upon by the same Gravity which will give it another Degree of Motion equal to each of the former, &c. so that in 1, 2, 3, 4, &c. Particles of Time the Body will acquire 1, 2, 3, 4, &c. Degrees of Motion, the Motion therefore of a falling Body is a Motion uniformly accelerated.

Q. E. D.
COR.

C O R. I.

Hence, and *per Axiom* 4th. the Motion of a Body thrown directly upwards is a Motion uniformly retarded: for the Force of Gravity, acting constantly and equally against the Motion commenc'd, will in equal Times equally diminish it 'till the whole Motion is destroy'd,

C O R. II.

A Body thrown directly upwards with the same Impetus, or Force, which it acquir'd by falling from a certain Height, will ascend to the same Height from which it fell; for Gravity will in equal Time destroy the Motion it formerly gave, and the Velocity will be the same in every Place of the Ascent that it was in the Descent, and since the Velocity and Time are equal in
both

of Heavy Bodies. 31

both Cases, the Spaces pass'd through will be equal.

THEOREM V.

The Space which a heavy Body falls through by its own Gravity, counting from the Beginning of the Time, is equal to half the Space it wou'd pass through in an equal Time, if it mov'd uniformly with the Velocity acquir'd at the End of the Fall.

DEMONSTRATION.

Let *AB* (*Fig. 4.*) represent the Time of the Fall, and *BC* at Right-Angles with it the Celerity acquir'd at the End of the Fall, draw the Diagonal *AC*, and through the Points 1, 2, 3, 4, &c. the Right-Lines, *1e*, *2f*, *3g*, &c. parallel to *BC*; because, by the last, the Celerities are every where as the Times elaps'd from the Beginning of the Fall,
those

32 *Of the Descent*

those Right-Lines, *1e*, *2f*, *3g*, &c. will represent the Celerities, in the Times, *o1*, *o2*, *o3*, &c. but since the Space pass'd over in each Particle of Time is (*per Schol. 2d.*) equal to the Rectangle of that Particle into its respective Celerity, if the whole Time *AB*, be supposed to be divided into indefinitely small equal Parts, the Sum of all those Rectangles, (which is equal to the whole Space pass'd through) may be consider'd as a Series of Lines of indefinitely small Breadth whose Sum constitute the Area of the Triangle

ABC, which is equal to $\frac{AB \times BC}{2}$, e-

qual to half the Rectangle of the whole Time *AB*, into the Celerity acquir'd at the End of the Fall : but if the Body had mov'd the whole Time *AB*, with the Celerity *BC*, the Space pass'd through wou'd (*per Schol. 2d.*) be equal to $AB \times BC$, which is double the former.

Q. E. D.
C O R.

C O R. I.

The Spaces pass'd over, reckoning from the Beginning of the Fall, are in the duplicate Ratio of the Times, or of the Celerities ; for the Space pass'd over in the Time AQ , is to the Space pass'd over in the Time AB , as the Triangle AQR , to the Triangle ABC : that is, (by 19. *El.* 6.) as AQ^2 , to AB^2 , or as the Square of QR , the Celerity acquir'd at the End of the Time AQ , to the Square of BC , the Celerity acquir'd at the End of the Time AB .

C O R. II.

Hence, both the Times and Celerities, are in the Sub-duplicate Ratio of the Spaces pass'd through by the falling Body.

E

C O R.

34 *Of the Descent*

C O R. III.

Hence, the Momenta, or Forces, which a Body acquires by falling from different Heights, are in the sub-duplicate Ratio of those Heights ; for (by *Law 4.*) the Momenta of equal Bodies are as the Celerities, and, by the last *Cor.* the Celerities are in the sub-duplicate Ratio of the Heights, or Spaces passed through, the Momenta are therefore in the sub-duplicate Ratio of those Heights.

C O R. IV.

Hence, and from *Cor. 2d.* to *Theo. 4.* the Forces requisite to throw equal Bodies to different Heights, are to each other as the Square Roots of those Heights.

DEFI-

DEFINITIONS.

I. The greatest Height that any Engine can throw a Body is call'd the Impetus of that Engine.

II. The Angle which the Line of Direction makes with the Horizon, is call'd the Elevation or Depression; according as it is above, or below the horizontal Line.

III. The Distance that a projected Body is thrown on any Plane is call'd the Amplitude, Range, or Randon.

C H A P. IV.

The DOCTRINE of PRO-
JECTILES.

T H E O R E M VI.

IF a Body is thrown, or projected, by any Force, or Engine, in any Direction not perpendicular to the Horizon, the Way, or Path, of that Body will be the Curve of the common Parabola.

D E M O N S T R A T I O N.

Let the Body be projected in the Direction *AI*, (*Fig. 5th.*) if no other
Force

of Projectiles. 37

Force impelled it, it wou'd continue to move equally in that Direction; (by *Law 1.*) but it is constantly sollicitated by the Force of Gravity towards the Earth in Right-Lines perpendicular to the Horizon, its Path must therefore be a Curve; (*Cor. 2d. Theor. 3d.*) and since its Progress is not alter'd with respect to the Distances parallel to the Directions of the moving Forces, (*Cor. 1st. Theor. 3d.*) the Points f, g, h, i , &c. in the Right-Lines, bf, cg, dh , &c. which are all in the Direction of the Force of Gravity, will all be arriv'd at in the same Time that the corresponding Points b, c, d , &c. wou'd be reach'd by the projecting Force only, and also in the same Time that the Points l, m, n, o , &c. in the Right-Lines lf, mg, nh , &c. parallel to the Direction of the projecting Force, wou'd be arriv'd at by the Body if no Force had acted upon it but the Force of Gravity only; but since the Motion

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on through AI , is an equable Motion, the Right-Lines $Ab, Ac, Ad, \&c.$ or their Equals, $lf, mg, nb, \&c.$ are as the Times in which the Body wou'd fall through the Spaces $bf, cg, db, \&c.$ or their Equals $Al, Am, An, \&c.$ but the Spaces passed through by a falling Body are as the Squares of the Times elaps'd from the Beginning of the Fall, (*Cor. 1st. Theor. 5th.*) therefore the Abscissæ or Parts of the Axis $Al, Am, An, \&c.$ are to each other as $lf^2, mg^2, nb^2, \&c.$ which is a known Property of the Abscissæ and Ordinates of the common Parabola.

Q. E. D.

C O R. I.

Hence, (and, from *Cor. 1st. to Theor. 3d.*) it is evident this Demonstration is general, whither the Direction of the Projectile Force be horizontal as AI , (*Fig. 5th.*) or obliquely ascending, or descending, as Ae , or AE . (*Fig. 6.*)

C O R.

C O R. II.

It's plain, from *Fig. 6*, that the Direction of any Force or Engine as *Ae*, is a Tangent to the Curve the projected Body describes, at the Point where the Engine is plac'd, for since Gravity acting every Instant, draws the Body from the Line of Direction as soon as it leaves the Engine, the Line of Direction can only touch the Curve at that Point.

C O R. III.

The Times of the Motion, or Flight, of the Body through different Parts of the Curve, *Af*, *Afg*, *Afgh*, &c. (*Fig. 6.*) are still as the corresponding horizontal Distances, *Az*, *Ay*, *Ax*, &c. for, by the last, the Times are as *Ab*, *Ac*, *Ad*, &c. that is, *per* Similarity, as *Az*, *Ay*, *Ax*, &c.

C O R.

C O R. IV.

The Times of the Flight of the same, or an equal Body projected by the same, or an equal Force, in different Directions, are as the corresponding Parts of the Lines of Direction, that is, the Time of the Flight to K , (*Fig. 5th.*) is to the Time of the Flight to i , (*Fig. 6th.*) as AI , in the first to Ae , in the Second.

THEOREM VII.

If a Body falls from any Height as KV , in the Right-Line KC (*Fig. 7 & 8.*) and if with the Velocity acquir'd at the Point V , the Direction be chang'd into VI , either horizontal, or oblique, so that the Body begins to move in a Parabola, if Vf , be taken equal VK , and, fD drawn parallel to VI , terminated by the Curve, $LD = 2fD$, will be

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be a third Proportional to any Abscissæ and its Ordinate, that is, let the Abscissæ VE , and Ordinate EG , be taken any where, it will still be $VE : EG :: EG : LD$.

DEMONSTRATION.

The Velocity acquir'd by the Fall through KV , wou'd in an equal Time carry the Body through $VB = 2KV$, in the Direction VI , if Gravity did not act; (*Theor. 5.*) but, Gravity beginning to act again at the Point V , the Body will in that Time be carried from the Line of Direction VI the Distance $BD = KV$, equal, by Construction, to Vf ; and because Vf and BD are equal and parallel, $VB = fD$; but $VB = 2KV$, therefore $fD = 2Vf$, & $\frac{1}{2}fD = Vf$; but (by *Theor. 6.*) Vf , or $\frac{1}{2}fD : fD^2 :: VE : EG^2$, consequently $\frac{1}{2}fD \times EG^2 = fD^2 \times VE$, and dividing by $\frac{1}{2}fD$, $EG^2 = 2fD \times VE$; but $2fD = LD$, there-
G fore

fore $EG^2 = LD \times VE$, which gives,
 $VE : EG :: EG : LD$. Q. E. D.

C O R. I.

This *Theorem* is universal whether the Direction of the falling Body be chang'd into a horizontal Direction as VI , (*Fig. 7th.*) or into an obliquely ascending, or descending Direction, as VI , or Vi . (*Fig. 8th.*)

C O R. II.

Hence, the Time of any Projection made on a horizontal Plane, is equal to double the Time of the Fall of a heavy Body from the Height of the Projection; for the Curve VgD (*Fig. 7.*) is describ'd by the projected Body in the same Time that it wou'd have fallen through its Height Vf by its own Gravity, and if, with the Velocity acquir'd at the Point D , the Body was projected di-

directly backwards, it wou'd arrive at the Point V in an equal Time with that of its Motion from it, and with an equal Velocity; for Gravity wou'd in equal Times decrease the Velocity as much as before it increased it, and if nothing prevented the Motion at the Point V it wou'd continue to describe an equal Curve VbL on the other Side; consequently the whole Curve $DgVbL$, wou'd be describ'd in double the Time in which the Body wou'd fall, by its own Gravity, through the Height of the Projection Vf .

DEFINITIONS.

I. The Right-Line LD is call'd the Parameter to the Diameter VC or Vertex V ; but if V is the highest Point in the Parabola, or principal Vertex, then

II. VC , the Axis, or principal Diameter.

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III. LD , the Latus-Rectum, or principal Parameter.

IV. VK , the Sublimity of the Parabola.

V. f , the Focus Point.

C O R. III.

All the Parabola's that can be describ'd by the same Impetus VK , will have their Parameters to the Vertex V , or Place of the Engine, equal; for let the Direction VI be what it will, LD the Parameter will still be equal to $4VK$, the Impetus wherewith the Parabola begins to be describ'd.

C O R. IV.

The Distance fV , from the Focus Point to the Vertex, is still equal to the Sublimity VK .

THEO-

THEOREM VIII.

If from the Focus Point f (Fig. 7th.) a Right-Line fG be drawn to any Point G in the Curve of a Parabola, it will be equal to the Height VE , of that Part of the Parabola, and Sublimity VK , that is, $fG = KE$.

DEMONSTRATION.

By the last $LD \times VE = EG^2$, and $EG^2 + Ef^2 = fG^2$; (47. *El.* 1.) but, because $LD = 2Kf$, and $\frac{1}{2}Kf + fE = VE$, $LD \times VE = 2Kf \times \frac{1}{2}Kf + fE = Kf^2 + 2Kf \times fE = EG^2$, to which add Ef^2 and you have $Kf^2 + 2Kf \times fE + fE^2 = EG^2 + Ef^2 = fG^2$ from above; but $Kf^2 + 2Kf \times fE + fE^2 = EK^2$, (4. *El.* 2.) Consequently $KE^2 = fG^2$, and $KE = fG$.
Q. E. D.

Note, Above the Focus Point f , the Demonstration will be the same, only it

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it will be -- $2 Kf \times fE$, because the Abscissæ Ve , is, in that Case, less than Vf the Distance from the Focus to the Vertex, as is easily observ'd by the Figure.

C O R. I.

Hence, if VK be the Sublimity of any Parabola, and if KR be parallel to the Ordinate EG , and GR parallel to the Axis VC ; GR , Gf , and EK will still be equal.

C O R. II.

Hence, if a Circle is describ'd on any Point G of the Curve with the Radius KE , or GR , equal to the Height and Sublimity, the Circumference of that Circle will still pass through the Focus Point of the Parabola.

C O R. III.

Hence, if one End of the Thread gEf , (*Fig. 15.*) equal in Length to the
Side

Side of the Square gD , be fix'd at g , and the other End at f , any where below the Rule AB , this being done, if you Slide the Side of the Square CD along the lower Side of the Rule AB , and, at the same Time, keep the Thread constantly tight by means of the Pin E , with its Part Eg close to the Side of the Square gD , the Curve EVG , which the Pin E describes by its Motion, will be a Parabola. *Note*, When the Side Dg touches the Point f , E will be at the Vertex, and then the Square must be turn'd to the other Side.

THEOREM IX.

If from any Point G (*Fig. 9th.*) in the Curve of a Parabola, a Right-Line Gf be drawn to the Focus Point f , and GR parallel to the Axis VC , and if the Angle fGR is bisected, the bisecting Line Tt will be a Tangent to the Curve in the Point G .

DE-

DEMONSTRATION.

Make VK , in the Axis produced, equal to Vf , and parallel to the Ordinate EG , draw KRD , then (by *Cor. 1st. Theor. 8th.*) KE , Gf and GR , are all equal; if therefore the Right-Line DH , parallel to GR , drawn to any other Point as H of the Right-Line Tt , be shorter than the Right-Line fH , drawn from the Focus to the same Point, that Point must be without the Curve; draw HR , since the Right-Line TGt bisects the Angle fGR , the Angles fGH and RGH are equal, and since $fG = RG$, and GH common to both Triangles fGH and RGH , the Side fH must be equal to the Side HR ; but DH being the Base of a Right-angled Triangle is shorter than RH the Hypothenuse, DH is therefore shorter than fH , and consequently the Point H is without the Curve;

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Curve; now, since it will be so let the Point H be taken where it will in the Right-Line Tt , except in the Point G , the Right-Line Tt , is therefore a Tangent to the Curve in that Point.

Q. E. D.

C O R. I.

The Distance fT , from the Focus to the Intersection of the Tangent and Axis produc'd, is equal to the Distance fG , from the Focus to the Point of Contact; for RG being parallel to Tf , and the Angle fGR being bisected, the Angle $RG T$, is equal to the Angle fTG , (by 29. *El.* 1.) equal to the Angle fGT ; the Triangle TfG is therefore Iosceles, (by 5. *El.* 1.) consequently $fT = fG$.

C O R. II.

The Distance VT , from the Vertex
to the Intersection of the Tangent and
H Axis,

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Axis, is equal to the Abscissæ VE , for KE , $(= fG) = Tf$, and Kf , common to both, being bisected in V , (*Cor. 4. Theor. 7.*) consequently $VT = VE$, Hence, $GE : 2VE :: R : T, EGT$.

THEOREM X.

The Velocity of a Body moving in the Curve of a Parabola will be, in any Point as G , (*Fig. 10.*) equal to what it wou'd acquire in falling through KE , $= KV + VE$ the Sublimity and Height of that Part of the Curve.

DEMONSTRATION.

If the Body was depriv'd of Gravity in any Point of the Curve as at the Vertex V , it wou'd continue to move equally in a Right-Line according to the Direction VI , and Velocity it had at that Point; (*per Law 1.*) in like Manner, if Gravity ceas'd to act in any other Point

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Point as G , the Body wou'd proceed equably in the Tangent GH , and (by *Cor. 1. Theo. 3.*) GB and HD being parallel to the Axis TC , or the Direction of the Force of Gravity, GH wou'd be passed through in this Case in the same Time that BD , or its equal GF , wou'd in the former Case; but by the similar Triangles $GFH, GET, GF:GH::GE:GT$, that is, the Velocity in the Vertex, is to the Velocity in any other Point in the Curve, as the Ordinate, is to the intercepted Tangent. Now (by *Theo. 7.*) $4VK \times VE = EG^2$, but $EG^2 + 4VE^2 (= ET^2) = GT^2$, (by 47. *El. 1.*) which two Equations give this Analogy, $GE^2:GT^2::4VK \times VE:4VK \times VE + 4VE^2$, and dividing the two last Terms by $4VE$, you have $EG^2:GT^2::VK:VK+VE$, or KE , consequently $GE:GT::\sqrt{VK}:\sqrt{VK+VE}$, or $\sqrt{KE}$; but EG , is to GT , as the Velocity in the Point V , is to the Velocity in the

H 2
Point

Point G , by what was demonstrated above, and the $\sqrt{VK}$, is to $\sqrt{KE}$, as the Velocity acquir'd by falling through VK , to the Velocity acquir'd by falling through KE , (*Cor. 2. Theo. 5.*) but the Velocity in the Point V , is equal to what wou'd be acquir'd by falling through KV , (*Theo. 7.*) consequently the Velocity in the Point G , is equal to what wou'd be acquir'd by falling through KE , the Height and Sublimity. $\mathcal{Q}. E. D.$

C O R. I.

If a Body falls through $ZA = KC$, the Height and Sublimity of the Parabola $AVGM$, and if with the Velocity acquir'd at the Point A , the Direction of its Motion be chang'd into At , the Tangent to the Parabola in that Point, it will continue to move in the Curve of that Parabola; for, (by *Theo. 6.*) it will move in a Parabola, and by the

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the last, the Velocity is the same where-
with that Parabola was describ'd, and
by Hypothesis, the Direction is a Tan-
gent to the Curve at that Point *A*, con-
sequently its Way, or Path, will be the
same upwards that it wou'd have been
downwards; for (*Cor. 2. Theo. 7.*) Gra-
vity will in equal Times destroy the Mo-
tion which before it gave, and by act-
ing always according to the same Te-
nor and Direction, it will retain the Bo-
dy in the same Curve.

C O R. II.

Hence, if a Body is projected di-
rectly upwards with the same Force
wherewith an equal Body is projected
obliquely, it will ascend to a Height e-
qual to the Height and Sublimity of
the Parabola describ'd by the Motion
of the oblique Projection.

C O R.

C O R. III.

Hence, every Parabola that can be describ'd by the same Impetus AZ , has its Height and Sublimity equal to that Impetus, for $AZ = CV + VK$.

C O R. IV.

All the Parabola's that can be describ'd by the same Impetus have their Focus Points $f, F, ff, \&c.$ (*Fig. 13.*) in the Circumference of a Circle, whose Radius is the Impetus AZ , and Center A , the Place of the Engine; for (by *Theo. 8. & Cor. ultima*) AZ, Af , and CK , are still equal let the Direction be what it will.

C O R. V.

All horizontal Amplitudes made, with the same Impetus at different Elevations,

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vations, are to each other as the Sines of the Arches Zf , ZF , &c. the Distances of the Focus from the Zenith of the Engine; for let the Focus Point f , be taken any where in the Semicircle $ZfFN$, its plain that Sf , the Sine of the Arch Zf , will still be equal to AC , the Ordinate to the Point A , which is half the Base or Amplitude Am ; but, because the Sine of the Arch Zf is also the Sine of fN , its Complement to a Semicircle, and because the Angle fAZ is double tAZ , the Complement of the Elevation, (*Theo.* 9.) the Arch fN , or Angle fAN , will still be double the Angle of the Elevation; for double any Angle and double its Complement make a Semicircle; Sf , is therefore still the Sine of double the Angle of Elevation.

C O R. VI.

Hence, all horizontal Randons made, with the same Impetus at different Elevations,

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vations, are to each other as the Sines of double the Angles of Elevation.

C O R. VII.

Hence also, the greatest horizontal Randon is double the Impetus, and the Elevation wherewith it is made is 45° . or half a Right-Angle; for when the Focus Point F , is in the horizontal Line, $AM = 2Af$, is the greatest horizontal Randon possible, by that Impetus; and because the Direction, or Tangent, bisects the Right-Angle FAZ , FAT the Elevation is half a Right-Angle.

C O R. VIII.

Hence, any Object, whose Distance on the Plane of the Horizon is less than the greatest Randon, may be hit by two Elevations, each of which is the Complement of the other to 90° . for when the Focus Points f , and ff , are equally

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qually above and below the horizontal Line AM , because the Sines fS and Sff are equal, the horizontal Amplitudes will be equal, (*Cor. 5th.*) and because the Directions still bisect the Angles between the Focus and Zenith, they must be equally above and below 45° , and such are still the Complement of each other to 90° .

C O R. IX.

In any two Parabola's describ'd by the same Impetus, and with Elevations equally above and below 45° , the Height of the one will be equal to the Sublimity of the other; for, because $fC = ffC$, $ffC + Cu = fC + Cu = uK$, (*per Theo. 7.*) and $uK + Cu = fC + 2Cu = CK$; but $fC + 2VK = CK$, (*Cor. 4th. Theo. 7th.*) therefore $fC + 2VK = fC + 2Cu$, and $Cu = VK$, that is Cu , the Height of the Parabola Aum = the Sublimity VK , of the Parabola AVm , &c.

I

THEO-

THEOREM XI.

The Time, or Duration, of any oblique Projection made on the Plane of the Horizon, is to the Time of a perpendicular Projection made with the same Impetus, as the Sine of the Elevation of the oblique Projection, to the Radius.

DEMONSTRATION.

Because $AZ = Af$, and the Direction At (*Fig. 10th.*) bisects the Angle fAZ , (*Theo. 9th.*) the Triangle AdZ is Right-angled at d ; and if AZ is made the Radius, Zd is the Co-sine, and Ad the Sine of the Angle of Elevation; make $AQ = CV$, and, because $fV = VK$, (*Cor. 4th. Theo. 7th.*) the Right-Line QV will pass through the Point d , and Triangle AQd is Right-angled at Q , therefore AZ , Ad , and AQ are
con-

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continual Proportionals, (by 8. *El.* 6.) and $AZ : Ad :: \sqrt{AZ} : \sqrt{AQ}$; but $\sqrt{AZ}$, is to $\sqrt{AQ}$, as the Time of the Descent of a heavy Body through AZ , to the Time of the Descent through AQ ; (*Cor.* 2. *Theo.* 5.) consequently those Times are as AZ , to Ad ; but the Time of every Projection is double the Time of the Fall from the Height of that Projection, (*Cor.* 2d. *Theo.* 7th.) therefore the Time of the oblique Projection is to that of the perpendicular Projection, as $\sqrt{AQ}$, to $\sqrt{AZ}$, or as Ad the Sine of the Angle of Elevation, to AZ the Radius.

Q. E. D.

C O R.

Since the Time of every oblique Projection is to that of the perpendicular Projection, as the Sine of the Angle of Elevation to the Radius, they are to each other as the Sines of their respective Elevations.

I 2

SCHOL-

SCHOLIUM III.

From what has been demonstrated, every *Problem* relating to Projections made on the Plane of the Horizon may be resolv'd by *Plain Trigonometry*; for, in the Triangle *AfC*, (*Fig. 13th.*)

I. $Af = AZ$ = the Impetus, or half the gr. Ran. (*Cor. 3d. Theo. 10th.*)

II. $AC = \frac{1}{2} Am = \frac{1}{2}$ the horizontal Amplitude. (*Cor. 5. Theo. 10.*)

II. $AfC = fAZ$ = double the Complement of the higher, or double the lower Elevation. (*Theo. 9. and Cor. 8. Theo. 10.*)

IV. $\frac{Af - fC}{2} = VK$ = the Sublimity of the Higher and Height of the Lower Projection, because $fV = VK = Cu$

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= Cu . (*Cor.* 4. *Theo.* 7. and *Cor.* 9. *Theo.* 10.)

$$V. \frac{Af + fC}{2} = uK = CV = \text{the Sub-}$$

limity of the lower and Height of the higher Projection, because each of these is the Complement of the other, to the Impetus Af . (*Cor.* 3. *Theo.* 10.)

Now, since the Right-Angle is always known, from any two of these five Articles being given, the rest may be found; but since the Angle fAN , or Arch fN , (*Fig.* 13.) is still double the Angle of Elevation, if, with the Radius AZ = to the Impetus, (*Fig.* 14.) the Semicircle $AZFN$ is describ'd, and its Periphery divided into 90 equal Parts, reckoning from N upwards, 10. 20. 30. 40. &c. to Z , the right Sines of those Parts $S10. S20. S30. S40. \&c.$ will still be half the horizontal Amplitudes at 10. 20. 30. 40. &c. Degrees of Elevation, and each respective ver-

fed

fed Sine SN , will be double the Altitude, and SZ , double the Sublimity of the Projection made at the corresponding Elevations.

THEOREM XII.

The greatest Distance that an Object can be reach'd on any Plane, as well ascending and descending, as horizontal, has the Focus Point of the Parabola describ'd by the projected Body in the Right-Line that joins the Object and Engine,

DEMONSTRATION.

Let the Impetus AZ , (*Fig. 11.*) and its equal YK , be perpendicular to the horizontal Line AM , and let AX be suppos'd to be the greatest Distance which that Impetus can reach on the ascending Plane AD , (by *Cor. 4. Theo. 10.*) all the Parabola's that can be describ'd

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scrib'd by that Impetus, have their Focus Points in the Circumference of the Circle $AZfN$; &c. and (by *Cor. 2. Theo. 8.*) the Parabola that passes through the Point X , has its Focus Point in the Circumference of the Circle $XKfY$, &c. describ'd on the Point X , with the Radius XK , equal to the Height and Sublimity of that Part of the Parabola. Now, it is plain if the Circles did not meet, the Point X wou'd be without the reach of that Impetus, and if they intersected each other, it's also obvious that the Center X might have been taken farther towards D , as in *Fig. 19th*, and then the Distance AX , as now taken, wou'd be less than the greatest Distance, which is contrary to the Hypothesis; the Circles must therefore only touch each other, and the Focus Point must be in the Point of Contact; but (by *11. El. 3d.*) the Point of Contact of any two Circles is in the Right-Line that joins their Centers,

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ters, that is, in the Right-Line AX that joins the Object X , and Engine A .
Q. E. D.

C O R. I.

After the same Manner it is demonstrated that the greatest Randon AX , (*Fig. 12.*) on any descending Plane, has the Focus Point f of the Parabola in the Line that joins the Object X , and Engine A , which is obvious by the Figure and what was said in the last.

C O R. II.

Hence, (and from *Theo. 9.*) the greatest Randon on any Plane, and consequently the least Force that can reach any Object, is with a Direction that bisects the Angle between the Object and Zenith.

C O R. III.

The greatest Randon that can be made by any Impetus on any ascending Plane,

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Plane, is double that Impetus less by the Height of the Plane, for $AX + XY = 2 Af$, and $2 Af - XY = AX$, (Fig. 11.)

C O R. IV.

Hence, if the Impetus Af , and Elevation of the Plane YAX are given, it will still be $R + S$, $YAX : R :: 2Af : AX$, for $R : S$, $YAX :: AX : XY$, and by Composition, (because $AX + XY = 2 Af$) $R + S$, $YAX : R :: 2Af : AX$.

C O R. V.

Hence, since R and $2 Af$ are still the middle Terms, let the Elevation of the Plane be what it will, the Product of the Distances on different Planes into the Radius more their respective Sines will still be equal, and consequently those Distances will be reciprocally

K cally

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cally proportional to the Radius more their Sines.

C O R. VI.

If the Impetus Af , and horizontal Distance AY , (*Fig. 11.*) of the Perpendicular YK are given, the greatest Height YX which that Impetus can reach on that Perpendicular will be =

$$Af - \frac{AY^2}{4Af}; \text{ since } AX + XY = 2Af,$$

let $a = XY$, and $2Af - a = AX$, then $AX^2 - XY^2 = AY^2$, (*47. El. 1.*) that is, $4Af^2 - 4Af \times a = AY^2$, which gives $a = Af - \frac{AY^2}{4Af} = XY$ the Height.

C O R. VII.

The greatest Randon AX , (*Fig. 12.*) that can be made on any descending Plane is double the Impetus Af , more YX the Depth, or Descent, of the Plane; for $AX = Af + XK$, that is, (because $Af = YK$) $AX = 2Af + YX$.

C O R.

C O R. VIII.

Hence, if the Impetus Af and Depression of the Plane YAX are given, it will still be $R - S, YAX : R :: 2 Af : AX$, for $R : S, YAX :: AX : XY$, and by Division, (because $AX = 2 Af + YX$) $R - S, YAX : R :: 2 Af : AX$.

C O R. IX.

Hence also, it's evident that if the Distance AX (*Fig. 19.*) is taken less than the greatest Distance the Impetus can reach, the Circles describ'd on A , and X , with the Radii Af and XK , will intersect each other in two Points f and F , equally distant from the Point b , where the Focus of the Parabola wou'd be if AX was the greatest Distance that Impetus cou'd reach; which Points f and F , will be the Foci of two Parabola's describ'd by that Impetus,

K 2 each

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each of which will pass through the Point X .

C O R. X.

Hence, any Object, whose Distance is less than the greatest the Impetus can reach, may be struck by two Directions each of which are equally above and below the Line that bisects the Angle between the Object and Zenith; for, since f and F , are equally above and below the Point b , the Direction which still bisects the Angle between the Zenith and Focus (*Theo. 9.*) must be equally distant from the Line that bisects the Angle between the Object and Zenith.

C O R. XI.

Hence, from the Impetus Af , (*Fig. 19.*) and Distance AX on a Plane of a given Ascent being given, the Direction

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on may be found, for from the given Distance AX , and Ascent YAX , the Height YX is given, then because $fX = Af - XY$, $2Af - XY$ is the Sum of the Sides $Af + fX$, and XY their Difference; whence in the Triangle AfX , it will be $AX : 2Af - XY :: XY : Aq$ the Difference of the Segments of the Base AX , and $\frac{AX + Aq}{2}$

$= AP$ the greater Segment; then $Af : AP :: R : S$, AfP ; but fd being perpendicular to AM the horizontal Line, and fP to AX , the Angle dfP , $= dAP$ the given Ascent, and $AfP - dfP = Afd = fAZ$ double the Complement of the Elevation: (*Theo. 9.*) therefore $90^\circ - \frac{1}{2} Afd =$ the Elevation. Note, if F the Focus is below the horizontal Line AM , $AFP + PFD = AFD$, or FAN will be double the Elevation. (*Cor. 5. Theo. 10.*)

C O R.

C O R. XII.

If the Impetus $AZ = Af$, (*Fig. 20.*) and Distance AX , on a Plane of a given Descent are given, to find the Direction, it will be $AX : 2 AF + XY ::$

$XY : XQ$, and $\frac{AX - XQ}{2} = AP$, then

$AF : AP :: R : S$, $AFP = AfP$, and $AFP - PFD = AFD = FAN$, double the Elevation; (*Cor. 5. Theo. 10.*)

whence $\frac{AFP - PFD}{2}$ equal the low-

er Elevation. And $AFP + PFD = AfP + Pfd$ equals Afd , double the Complement of the higher Elevation.

Whence $90^\circ - \frac{1}{2} Afd =$ the higher Elevation, all which is plain from the Figure and what was said in the last.

Note, If the higher Direction exceed the Complement of the Descent of the Plan XAN , the lower Direction will be a Depression, which is plain from the

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the Figure ; for in that Case the lower Focus will fall beyond the Nadir Point N , as at ff , and because the Direction still bisects the Angle between the Focus and Zenith, it will be below the horizontal Line AM , but still ffN , or Angle $ffAN$, will be double the Depression.

$L E M M A \ I.$

In every Parabola, the Latus-Rectum $L = LD$ (*Fig. 7.*) is to the Sum of any two Ordinates, as the Difference of those Ordinates, is to the Difference of their Abscissæ, that is, $L : AY :: YM : YG$.

$DEMONSTRATION.$

$L \times VC = AC^2$, and $L \times VE = EG^2$, (*Theo. 7.*) whence, $L \times VC - L \times VE = AC^2 - EG^2$, consequently, $L : AC + EG :: AC - EG : VC - VE$; that is, (because $AC + EG = AY$, and $AC - EG = YM$, also, $VC - VE = YG$) $L : AY :: YM : YG$. Q. E. D.
 $L E M-$

L E M M A II.

The Latus-Rectum L , is to any double Ordinate AM , (*Fig. 21.*) as Radius to the Tangent of the Angle $MA t$, which that double Ordinate makes with the Tangent At to the Point A .

D E M O N S T R A T I O N.

Let $b = AM$, $r =$ the Radius, $b =$ the Height CV , and T the Tangent; then $\frac{1}{2}b : 2b :: r : T$. but $b : 4b :: L : b$, (*Theo. 7.*) consequently, $L : b :: r : T$.

Q. E. D.

T H E O R E M XIII.

In any Triangle inscrib'd in a Parabola having a double Ordinate AM (*Fig. 21.*) for its Base, and its Vertex x any where in the Curve above that Ordinate,

dinate, the Sum of the Tangents of the Angles at the Base, will still be equal the Tangent of the Angle $MA t$, made by the Tangent to the Point A and its Ordinate.

DEMONSTRATION.

Let $z = AY$, and L, b, r and T , as before: then, (*per Lemma 1.*) $L : z ::$

$$b - z : Yx, = \frac{bz - zz}{L}, \text{ and (per Fig.)}$$

$$z : \frac{bz - zz}{L} :: r : T, YAx, \text{ equal}$$

$$\frac{rb - rz}{L}; \text{ but, because } b - z = YM,$$

$$b - z : \frac{bz - zz}{L} :: r : T, YMx, \text{ equal}$$

$$\frac{zr}{L}; \text{ but the Sum of these Tangents}$$

$$\frac{rb - rz}{L} + \frac{rz}{L} = \frac{rb}{L}, \text{ equal (per Lem.}$$

2.) the Tangent of the Angle $MA t$.

Q. E. D.

L

COR.

C O R. I.

Hence, from the Direction and horizontal Amplitude given, the Distance Ax on a Plane of any given Ascent is found: for $T, MAx - T, MAx = T, AMx$, and having the Angles at the Base you have the Angle AxM , then $S, AxM : S, AMx :: AM : Ax$. the Distance requir'd.

C O R. II.

Hence, from the Direction and Distance on a Plane of a given Ascent, the horizontal Amplitude is found.

C O R. III.

Hence also, from the horizontal Amplitude and Distance on a Plane of a given Ascent being given, the Direction may be found: for in the Triangle
 AxM

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AxM the Sides AM , Ax , and the Angle MAx , are given to find the Angle AMx ; then, $T, MAx + T, AMx = T, MAx$ the Elevation sought.

THEOREM XIV.

The Difference of the Tangents of the Angles MAx , and RXT , which any two Tangents make with their respective Ordinates in opposite Sides of the Parabola, is still equal to double the Tangent of the Angle AXR , or MAX , which the Right-Line AX , that joins the Points of Contact, makes with either Ordinate.

DEMONSTRATION.

Let $XN = z$ and L, b, r and T , as before; then, because $RN = z - b$ and $2z - b = RX$, (*per Fig. 21. and Lem. 2.*) $L : 2z - b :: r : T, RXT, =$

$L \quad 2$
 $2rz$

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$\frac{2rz - rb}{L}$; but (*per Lem. 1.*) $L : z ::$

$z - b : AN = \frac{zz - bz}{L}$ and (*per Fig.*)

$z : \frac{zz - bz}{L} :: r : T, AXR = \frac{rz - br}{L}$.

double which (*viz.*) $\frac{2rz - 2rb}{L}$, ta-

ken from $\frac{2rz - rb}{L} = T, RXT$ found

above, leaves $\frac{br}{L} = (\textit{per Lem. 2.}) T,$

Mat.

Q. E. D.

C O R. I.

. Hence, if the Elevation *Mat* and horizontal Amplitude *AM* are given, the Distance *AX* on a Plane of any given Descent *MAX*, may be found; for (by the 13th. and last) $T, RXA + T, XRA$, and $T, Mat + 2T, MAX$
($= 2T,$

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($= 2T, AXR$) are each equal to T, RXT , wherefore $T, MA t + T, MAX = T, XRA$, or RXM , and $RXM - RXA = AXM$; consequently, in the Triangle MAX the three Angles and the Side AM , are given to find the Side AX .

C O R. II.

The Elevation $MA t$ and Distance AX , on a Plane of a given Descent, being given, the horizontal Amplitude AM may be found: for the $T, MA t + T, MAX = T, XRA$, or RXM , and $RXM - RXA = AXM$: consequently, in the Triangle MAX the three Angles and the Side AX , are given to find the Side AM .

C O R. III.

If the horizontal Amplitude and Distance on a Plane of a given Descent
are

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are given, the Direction may be found: for in the Triangle AXM the Sides AM and AX , and their included Angle are given, to find the Angle AXM ; then, the Tangent of the Sum of the Angles $AXM + XAM$, less by the Tangent of the Angle $XAM =$ the Tangent of MA , the Elevation requir'd.

The End of the First Part:

PART

PART II.

The DOCTRINE of PROJECTILES *apply'd to* PROBLEMS of PRACTICAL GUNNERY.

HAVING in the foregoing *Sections* explain'd and demonstrated the *Theory of Projectiles*, and the *Laws of Motion and Gravitation*, as far as was requisite to the Knowledge of it; I now proceed to apply that *Theory* to the *Art of Gunnery* in all the most useful Examples that I think an *Ingeneer* can have Occasion for, with respect to the projectile Part, which is all that is design'd here; but first, it will be convenient to shew how to direct a Piece of Ordnance according to any Degree of Elevation or Depression.

Let

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Let AB , (*Fig. 18.*) be a Ruler of about five Foot long, and let each Quadrant of the Semicircle ADC , qDC be divided into 90° , and mark'd at each ten, thus, 10, 20, 30, &c. counting from D towards A , and q , the Spring Bfg is made of Steel and fixt to the End B , in order to keep the Ruler close to the lower Side of the Cylinder, or Bore of the Piece, that so it may be parallel to the Axis of the Cylinder when it is thrust into it: Now it is plain, by the *Fig.* that if the Piece is elevated any Number of Degrees from the horizontal Line HO , the Thread of the Plummet CK , will cut the Number of Degrees on the Limb of the Quadrant, for the Angle $CBO = KCD$, because BCK is the Complement of each to a Quadrant. And thus may a Mortar, or any other Piece of Ordnance, be elevated or depress'd to any Degree, which is call'd laying them to pass.

C H A P.

CHAP. I.

*Of PROJECTIONS made on
the PLANE of the HORIZON.*

CASE I.

A Piece whose greatest Randon is AM (*Fig. 13.*) = 8000, Quere her Elevation to strike an Object at m , whose Distance from the Piece is $Am = 5280$; as also the Height CV , and Sublimity VK , of the Projection.

$\frac{1}{2} AM = Af = 4000$, $\frac{1}{2} Am = AC = 2640$, then, $Af : AC :: R : S$, $AfC = 41^{\circ}. 18'$, and $90^{\circ} - \frac{1}{2} AfC = 69^{\circ}. 21'$, the higher Elevation, also $\frac{1}{2} AfC = 20^{\circ}$.

M 39'

39' the lower Elevation, and, $R : S, C$
 $Af :: Af : fC = 3005, \frac{Af + fC}{2} = CV$
 $= 3502.5$ the Height, also $Af - VC$
 $= VK = 497.5$ the Sublimity; all which
 is obvious from the *Figure* and *Scho-*
lium 3d.

C A S E II.

A Piece whose greatest Randon is
 AM (*Fig. 13.*) = 8000, Quere her
 Randon at $69^\circ. 21'$, = $CA t$, as also,
 the Height and Sublimity of the Pro-
 jection.

$R : S, 2 | \overline{CA t} :: AM : Am = 5280.$
 the Distance required; (*Cor. 6. Theo.*
10.) the rest are found, as *per Case 1.*

C A S E III.

The greatest Randon of a Piece be-
 ing AM (*Fig. 13.*) = 8000, and Height
 of

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of a Projection made at Random $CV = 3502.5$ given, to find the Sublimity, Direction, and Amplitude.

$\frac{1}{2} AM = Af = CK$, and $CK - CV = VK = 497.5$, the Sublimity, and $CV - VK = fC = 3005$; (*Cor. 4. Theorem 7.*) then $Af : fC :: R : S$, $Caf = 48^\circ.42'$, and $\frac{90 + Caf}{2} = 69^\circ.21' = CA t$, the Direction sought; also $R : S, 2 \overline{CA t} :: AM : Am = 5280$, the Amplitude.

CASE IV.

The greatest Random AM (*Fig. 13.*) = 8000, and Sublimity of a Projection $VK = 497.5$ given, to find the Height CV , Elevation $CA t$, and Amplitude AM .

$\frac{1}{2} AM = Af$, and $Af - VK = CV = 3502.5$ the Height, and (by *Cor. 4. Theo. 7.*) $CV - VK = fC = 3005$, and
M 2 having

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having Af and fC , the rest are found, as *per Case 3*.

C A S E V.

The Amplitude Am (*Fig. 13.*) = 5280, and Elevation CAt equal $69^\circ. 21'$ given, to find the Impetus Af , Height CV , and Sublimity VK .

$S, 2 | \overline{CAt} : R :: Am : AM = \text{gr. Ran.} = 8000$, and $\frac{1}{2} \text{ gr. Ran.} = Af = 4000$ the Impetus; (*Cor. 6 & 7. Theo. 10.*) then, $2CAt - 90^\circ = CAf$. *per Fig.* and $R : S, CAf :: Af : fC = 3005$; also (*per Schol. 3.*) $\frac{Af + fC}{2} = CV = 3502.5$ the Height, and $Af - CV = VK = 497.5$ the Sublimity of the Projection.

C A S E VI.

The Amplitude Am (*Fig. 13.*) = 5280, and Height $CV = 3502.5$ given,

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ven, to find the Elevation CAt , Impetus Af , and Sublimity VK .

$\frac{1}{2} Am = 1640 = AC$, and $2 CV = Ct$; (*Cor. 2. Theo. 9.*) then $AC : Ct :: R : T$, $CAt = 69^\circ. 21'$ the Direction sought, and having the Amplitude Am , and Direction CAt , the rest are found, as *per Case 5.*

C A S E VII.

The Amplitude Am (*Fig. 13.*) = 5280, and Sublimity $VK = Cu = 497.5$ the Height of the lower Projection (*Cor. 9. Theo. 10.*) being given, to find the Direction CAt , Impetus Af , and Height CV .

$\frac{1}{2} Am = AC$, and $2 Cu = Ci$, (*Cor. 2. Theo. 9.*) then $AC : Ci :: R : T$, $CAi = 20^\circ. 39'$, and $90^\circ - CAi = 69^\circ. 21' = CAt$ the Direction sought, (*Cor. 8. Theo. 10.*) and having the Direction
and

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and Amplitude, the rest are found as
per Case 5.

C A S E VIII.

The Elevation $CA t = 69^{\circ}.21'$ (*Fig. 13.*) and Height $CV = 3502.5$ being given, to find the Amplitude Am , and Sublimity VK .

$2CV = Ct$, (*Cor. 2. Theo. 9.*) and $S, CA t : S, Ct A :: Ct : CA = 2640$, and $2CA = Am = 5280$ the Amplitude, the rest are found as *per Case 5.*

C A S E IX.

The Elevation $CA t$ (*Fig. 13.*) = $69^{\circ}.21'$, and Sublimity $VK = 497.5 = Cu$ the Height of the lower Projection (*Cor. 9. Theo. 10.*) being given, to find Am , Af and CV , the Amplitude, Impetus, and Height.

90°

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$90^\circ - CAt = 20^\circ. 39' = CAi$, the lower Elevation, (*Cor. 8. Theo. 10.*) and $2Cu = Ci$. (*Cor. 2. Theo. 9.*) then $S, CAi : S, CiA :: Ci : CA = 2640$, and $2AC = Am = 5280$, the Amplitude sought, also $S, 2 \overline{AiC} : AC :: R : Af = 4000$ the Impetus, and $Af - VK = CV = 3502.5$ the Height of the Projection.

C A S E X.

The Height $CV = 3502.5$, and Sublimity $VK = 497.5$ given, to find the Impetus Af , Elevation CAt , and Amplitude Am . (*Fig. 13.*)

$CV + VK = CK = Af$, the Impetus (*Cor. 3. Theo. 10.*) $CV - VK = 3005 = fC$, (*Cor. 4. Theo. 7.*) then $Af : fC :: R : S, fAC = 48^\circ. 42'$, and $\frac{90^\circ + fAC}{2} = 69^\circ. 21' = CAt$ the Elevation, also $R : S, 2 \overline{CAi} :: 2 Af : Am = 5280$ the Amplitude sought.

Note,

Note, If nothing is required of the *Ingeneer* but to reach the Object, let 45° be the constant Elevation, and the absolute Forces which are as the square Roots of the Impetuses (*Cor. 4. Theo. 5.*) are as the square Roots of the Distances; (*Cor. 7. Theo. 10.*) but the absolute Forces are as their adequate Causes, the Gun-Powder, whence the Charges requisite to carry equal Shot different Distances, are as the square Roots of those Distances.

C A S E XI.

The greatest Randon of a Cannon-Royal being 8000 Paces, and her Requisite of Powder for that Randon 28 Pounds, Quere the Requisite of Powder to carry her Shot 3600 Paces.

$\sqrt{8000} : \sqrt{3600} :: 28 : 18\frac{2}{3}$, the Charge required. *Note*, If all the Powder exerted its Force in the Chace, or Bore, of the Piece, this Proportion wou'd

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wou'd be Mathematically true; but some of it, especially in high Charges, does not take Fire till out of the Mouth of the Piece, which will occasion some Variation, &c. By Randons made at this Elevation much Gun-Powder may be sav'd, and the Ingeneer more certain of hitting the Object than at any other; for an Error of a Degree above, or below 45° , will not produce a sensible one in the Distance, because the Sines within a Degree of 90° , differ very little from the Radius, and those Sines near the Elevation of 45° , are as the Distances.

C A S E XII.

The Impetus of a Cannon Royal being 12000 Feet, Quere the Celerity of her Shot at parting from the Mouth of the Piece, or at meeting with the Object on the Plane of the Horizon, for in both Cases, the Celerity is equal

N to

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to what wou'd be acquir'd by falling through the given Impetus. (*Theo.* 10.)

$8 \times \sqrt{12000} = 876$ the Celerity required, or the Feet the Ball moves through in a Second of Time: this follows from *Theo.* 5. and its *Corolaries*, and because a heavy Body falls through 16 Feet in a Second of Time. For, from these $\sqrt{16} : 32 :: \sqrt{12000} : 876$ the Celerity, and because 4, and 32 are the two first Terms in the Ratio, the square Root of any Impetus multiply'd by 8, will give the Celerity.

C A S E XIII.

The Impetus of a Piece being 12000 Feet, and the Weight of her Shot 63 Pounds, Quere the absolute Force, or Momentum of the Ball in known Weight.

By the last Case, $8 \times \sqrt{12000} = 879 = C$ the Celerity, and (*per Scholium 2.* to the Laws of Motion) $B \times C \times 1.3 = M,$

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M, that is, in Numbers, $63 \times 876 \times 1.3 = 71744 =$ the Momentum or absolute Force in Pounds, which is about $32\frac{1}{2}$ Tuns *Averdupois*. *Note*, because 8, and 1.3 are constant Multipliers, $8 \times 1.3 = 10.4$ will be a constant Multiplier.

C A S E XIV.

A Piece whose greatest Randon. is 8000, and whose absolute Force is 71744 Pounds, Quere with what Force she will strike an Object whose Distance is 5600, and whose Surface reclines, or leans directly from the Perpendicular 42° .

$8000 : 5600 :: R : S, 44^\circ. 26'$ double the Elevation, consequently the Ball strikes the Plane of the Horizon at an Angle of $22^\circ. 13'$, and the Surface of the Object, which reclines from the Perpendicular 42° , at an Angle of $70^\circ. 13'$; then (*per Theo. 2.*) $R : S, 70^\circ.$

N 2
13'

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13' :: 71744 : 67512 the Force required.

C A S E XV.

The greatest Randon of a Piece being 8000, Quere at what Distance she will strike an Object with the greatest Force possible whose Surface reclines 42° .

Since the Surface reclines 42° , it's plain, the Direction to strike it at Right Angles, which is the Direction of the greatest Force, is 42° ; then $R ; S, 2$
 $| 42^\circ :: 8000 : 7956$, the Distance required. Hence it's easy to observe, that if the Surface of an Object recline 45° , the greatest Force it can be struck with is from the greatest Distance, and the contrary.

Note, If the Ingeneer has Field-Room, and can vary his Positions and Distances as he pleases, he may strike
 an

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an Object, whose Surface is in a given Position, at any Angle, and consequently with any Force he pleases, not exceeding the greatest Force of his Piece.

C A S E XVI.

The greatest Randon of a Piece being 8000, Quere at what Distance she will strike an Object, whose Surface reclines 42° , with a Force which shall be to the greatest Force of the Piece as 3 to 4.

$4 : 3 :: R : S$, $48^\circ. 35' =$ the Angle the Object must be struck at, (*Theo.* 2.) and since its Surface reclines 42° from the Perpendicular, its Complement 48° , taken from $48^\circ. 35'$ leaves $35'$ for the lower Elevation, and $48^\circ. 35' - 42^\circ = 6^\circ. 35'$ the Complement of $83^\circ. 25'$, the higher Elevation. Then $R : S$, $1^\circ. 10' :: 8000 : 163$, the Distance at
the

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the lower Elevation, and $R : S, 2$
 $\sqrt{80^\circ.25'} :: 8000 : 1822$ the Distance
 at the higher Elevation.

C A S E XVII.

A Piece whose gr. Ran. with 28
 Pounds of Powder is 8000, Quere her
 Charge to strike an Object at Right-
 Angles whose Distance is 1822, and
 whose Surface reclines 42° .

It's plain, the Direction which strikes
 this Object at Right-Angles must be
 an Elevation of 42° . Then $S, 2 \sqrt{42^\circ} :$
 $R :: 1822 : 1832$ = the greatest Ran-
 don for the Charge required, and
 $\sqrt{8000} : \sqrt{1832} :: 28 : 13.3$ the Charge
 requir'd.

C H A P. II.

Of Projections made on Ascending Planes, and to Perpendicular Heights.

C A S E XVIII.

THE greatest Randon of a Piece on the Plane of the Horizon being 8000, Quere her greatest Randon on a Plane whose Ascent is $6^{\circ}. 30'$.

$R + S, 6^{\circ}. 30' : R :: 8000 : 7186.4$
 the Randon required. (*per Cor. 4. Theo. 12.*) *Note*, The most exact and expeditious Way to perform this and the like Operations is by the natural Sines ;
 for

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for $R + S$, $6^\circ. 30' = 11132032$, and
 $8000 \times R = 8000000000000$, then
 $11132032)8000000000000(7186.4$;
 but if any one choose to do it by *Logarithms*, he must strike off 4 Figures
 of the first Term for Decimals, and
 take 3 for the Log. of the Radius,
 then will all the Terms be had in the
 common Tables.

C A S E XIX.

The greatest Randon on a Plane
 whose Ascent is $6^\circ. 30'$ being 7186.4,
 Quere the greatest Randon on the
 Plane of the Horizon.

$R : R + S, 6^\circ. 30' :: 7186.4 : 8000$
 the Randon required.

C A S E XX.

The greatest Randon on a Plane
 whose Ascent is $6^\circ. 30'$ being 7186.4;
 Quere

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Quere her greatest Randon on a Plane whose ascent is $10^{\circ}. 20'$.

By *Cor. 5. Theo. 12.* $R + S, 10^{\circ}. 20'$
 $: R + S, 6^{\circ}. 30' :: 7186.4 : 6783$, the
 Randon required.

C A S E XXI.

The greatest Randon of a Piece on the Plane of the Horizon being 8000, Quere the greatest Height she can reach on the Perpendicular YK (*Fig. 11.*) whose horizontal Distance is AY , $= 7143$.

By *Cor. 6. Theo. 12.* $\frac{1}{2}$ gr. Randon —
 $\frac{AY^2}{\frac{1}{2} \text{ gr. Ran.}} = XY = 811$ the Height
 sought. Or by the *Logarithms* thus,
 from the double Log. of AY , take the
 Logarithm of double the gr. Randon
 the Number answering the Remainder
 taken from $\frac{1}{2}$ the gr. Ran. is the Height
 required.

Q

CASE

C A S E XXII.

The greatest Randon on the Plane of the Horizon being 8000, Quere the Direction to strike the greatest Height possible on the Perpendicular YX , (*Fig. 11.*) whose horizontal Distance is $AY = 7143$.

Find YX , that greatest Height by the last *Case*, then $XY : AY :: R : T$, $AXY = T, 83^{\circ}.31' = T, XAZ$, and $90^{\circ}.$ — $\frac{1}{2}XAZ = 48^{\circ}.15'$, the Direction required.

C A S E XXIII.

A Piece whose horizontal Randon at the Elevation $YAI = 64^{\circ}.02'$ (*Fig. 19.*) is $Am = 6060$, Quere her Randon at the same Elevation on the Plane AX , whose Ascent is $YAX = 6^{\circ}.30'$.

$T, YAI - T, YAX = T, AmX = T$, $62^{\circ}.44'$, (*Cor. 1. Theo. 13.*) whence
 YAX

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$YAX + AmX = 69^{\circ}. 14'$, which gives $AXm = 110^{\circ}. 46'$, but the Sine of the Sum of any two acute Angles is always equal to the Sine of the obtuse Angle, therefore $S, 69^{\circ}. 14' : S, 62^{\circ}. 44' :: Am : AX = 5761$ the Distance required.

CASE XXIV.

A Piece whose Randon at $64^{\circ}. 02' = YAI$, is $AX = 5761$, on a Plane whose Ascent is $YAX = 6^{\circ}. 30'$, Quere her Ran. on the Plane of the Horizon at the same Elevation.

$T, YAI - T, YAX = T, AmX = T, 62^{\circ}. 44'$, then $S, AmX : S, AXm :: AX : Am = 6060$.

CASE XXV.

A Piece whose Randon at 40° . on the Plane of the Horizon is AM (Fig. O 2 19.)

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19.) = 7580, Quere her Ran. at $64^{\circ}.02'$,
 $02' = YAI$, on a Plane whose Ascent
 is $YAX = 6^{\circ}.30'$.

$S, 2 \mid 40^{\circ} : S, 2 \mid 64^{\circ}.02' :: AM :$
 $Am = 6060$, then, $T, YAI - T, YAX$
 $= T, AmX = T, 62^{\circ}.44'$, lastly $S, AXm :$
 $S, AmX :: Am : AX = 5761$.

CASE XXVI.

A Piece whose Randon at $64^{\circ}.02' =$
 MAI (Fig. 23.) is $AX = 5761$, on a
 Plane whose Ascent is $MAX = 6^{\circ}.30'$,
 Quere her Randon on the same Plane
 at $21^{\circ}.09' = MAi$.

$T, MAi - T, MAX = T, AMX$,
 whence we have AXM , then $S, AMX :$
 $S, AXM :: AX : AM = 6060$, and
 $S, 2 \mid MAI : S, 2 \mid MAi :: AM : Am$
 $= 5180$, also $T, MAi - T, MAX =$
 $T, Amx = T, 15^{\circ}.16'$; lastly, $S, Axm :$
 $S, Amx :: Am : Ax = 3678$, the Ran-
 don required.

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CASE XXVII.

A Piece whose Randon at $m Ai = 21^{\circ}.09'$, on a Plane elevated $m Ax$, or $MAX = 6^{\circ}.30'$, is $AX = 3678$, Quere her Randon at $MAI = 64^{\circ}.02'$, on a Plane whose Ascent is $MAD = 13^{\circ}$,

$T, m Ai - T, m Ax = T, Amx = T, 15^{\circ}.16'$, whence we have Axm , then $S, Amx : S, Axm :: Ax : Am = 5180$. and $S, 2 | m Ai : S, 2 | MAI :: Am : AM = 6060$; also $T, MAI - T, MAD = T, AMD$, whence we have ADM , as before, and $S, ADM : S, AMD :: AM : AD = 5533$ the Randon required.

CASE XXVIII.

A Piece whose Randon at $MAI = 64^{\circ}.02'$ (*Fig. 19.*) on the Plane of the Horizon is $AM = 6859$, Quere how high she will strike an Object with the same

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same Direction on a Perpendicular YK , whose horizontal Distance is $AY = 6478$.

$T, MAI : R :: AM : L = 3340 =$ the Latus Rectum ; (*Lem. 2.*) then $AM - AY = 381 = YM$, and $L : AY :: YM : YX = 739$ (*Lem. 1.*) the Height required.

CASE XXIX.

A Piece at A (*Fig. 19.*) elevated $MAI = 64^\circ. 02'$, strikes an Object at X , whose perpendicular Height above the Plane of the Horizon is $YX = 739$, and whose horizontal Distance is $AY = 6478$, Quere her Randon on the Plane of the Horizon with the same Elevation.

$AY : YX :: R : T, YAX = T, 6^\circ. 30'$, and $T, MAI - T, YAX = T, AMX = T, 62^\circ. 44'$, whence $YXM = 27^\circ. 16'$; then $S, AMX : S, YXM :: YX : YM = 381$.

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= 381. and $AY + YM = AM = 6859$
the Randon required.

C A S E XXX.

A Piece whose horizontal Amplitude at $64^{\circ}.02'$ is AM (*Fig. 7.*) = 6859, strikes an Object with the same Direction on the Top of the Perpendicular $YG = 739$. Quere the horizontal Distance AY , or AO of that Perpendicular.

$R : T, 64^{\circ}.02' :: AC (= \frac{1}{2}AM) : 2CV$,
(*Cor. 2. Theo. 9.*) whence $CV = 3521$,
and $CV - YG = VE = 2782$, and (*per Theo. 6.*) $CV : CM^q :: EV : EG^q$,
whence $EG = CY = 3048.5$, and $AC + CY = 6478$, the greatest Distance,
and $AC - CY = AO = 381$, the least Distance.

C A S E XXXI.

A Piece at A (*Fig. 19.*) elevated
in $AI = 64^{\circ}.02'$ strikes an Object at X
whose

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whose Perpendicular Height above the Plane of the Horizon is $YX = 739$, and whose horizontal Distance is $AY = 6478$, Quere her Direction to strike an Object on the Plane of the Horizon whose Distance is $AM = 7812$.

$AY : YX :: R : T$, $YAX = 6^\circ. 30'$, and $T, mAI - T, YAX = T, YmX = T, 62^\circ. 44'$, whence we have $YXm = 27^\circ. 16'$, then $S, YmX : S, YXm :: YX : Ym = 381$, and $AY + Ym = Am$, lastly $Am : AM :: S, 2 \overline{mAI} : S, 63^\circ. 42'$, $\frac{1}{2}$ of which $31^\circ. 51' =$ the Direction required.

CASE XXXII.

A Piece whose gr. Ran. on the Plane of the Horizon is 8000, Quere her Direction and greatest horizontal Distance AY (*Fig. 11.*) from which that Piece can strike an Object on the Top of the Perpendicular $YX = 2179$.

gr.

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gr. Ran. $-YX = 5821 = AX$, (*Cor.*
3. Theo. 12.) and $AX : XY :: R : S$,
 $YAX = 21^{\circ}.58'$, whence $AXY =$
 $68^{\circ}.02'$, and $90^{\circ} - \frac{1}{2}AXY = 55^{\circ}.59'$
 $= YAI$, the Direction, (*Cor. 2. Theo.*
12.) then $R : S, YXA :: AX : AY$
 $= 5398$ the Distance sought.

CASE XXXIII.

The Perpendicular Height of an
Object $XY = 2179$, and horizontal
Distance $AY = 5398$ being given, to
find the least Impetus that possibly can
reach that Object.

$\sqrt{AY^2 + XY^2} = 5821 = AX$, (*47.*
El. 1.) and $\frac{AX + XY}{2} = 4000 = AZ$
the Impetus required. (*Cor. 3. Theo. 12.*)

CASE XXXIV.

A Piece whose Impetus is Af , (*Fig.*
19.) $= 4000$, Quere her Direction to
P strike

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strike an Object whose Distance is $AX = 5761$ on a Plane whose Ascent is $YAX = 6^\circ. 30'$.

$R : S, YAX :: AX : YX = 652$, and
(*Cor. 11. Theo. 12.*) $AX : 2 Af - XY$
 $:: XY : Aq = 831$, then $\frac{AX + Aq}{2} =$

$AP = 3296$, and $Af : AP :: R : S$,
 $AfP = 55^\circ. 30'$, also $AfP - YAX$
 $Afd = fAZ$, and $90^\circ - \frac{1}{2} fAZ = 65^\circ.$
 $30' = YAI$ the higher, and $\frac{1}{2} fAZ =$
 $24^\circ. 30'$ the lower Elevation. (*Ar. 3.*
Scholium 3.)

CASE XXXV.

A Piece whose Randon at $32^\circ. 28'$,
on a Plane whose Ascent is YAX ,
(*Fig. 19.*) $= 6^\circ. 30'$, is $AX = 6520$,
Quere her Direction to strike an Ob-
ject whose horizontal Distance is 6000.

$T, 32^\circ. 28' - T, 6^\circ. 30' = T, 27^\circ. 34'$
 $= T, AmX$, whence $AXm = 145^\circ.$
 $56'$

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56', then $S, AmX : S, AXm :: AX : Am = 7892$, the horizontal Amplitude at $32^\circ. 28'$; lastly $7892 : 6000 :: S, 2 | 32^\circ. 28' : S, 43^\circ. 32'$, $\frac{1}{2}$ of which $= 21^\circ. 46'$ the Direction required.

C A S E XXXVI.

A Piece whose Impetus is Af (Fig. 19.) $= 4000$, Quere her Direction to strike an Object on the Top of a Perpendicular whose Height is $YX = 652$, and whose Distance, in a straight Line, is $AX = 5761$.

$AX : XY :: R : S, YAX = 6^\circ. 30' = dfP$, (Cor. 11. Theo. 12.) and $AX : 2Af - YX :: YX : Aq = 831$, also $\frac{Af + Aq}{2} = AP$, then $Af : AP ::$

$R : S, AfP$, and $AfP - dfP = Afd = fAZ$; lastly, $90^\circ. - \frac{1}{2}fAZ = 74^\circ. 41'$ the Direction sought. (Ar. 3. Scho- lum 3.)

P 2

CASE

C A S E XXXVII.

A Piece whose horizontal Amplitude is AM (*Fig. 19.*) = 6859, strikes an Object with the same Direction on the Top of a Perpendicular whose Height is $YX = 739$, and whose Horizontal Distance is $AY = 6478$, Quere that Direction.

$Am - AY = Ym = 381$, then, (*Lem. 1.*) $YX; Ym :: YA : L = 3340$, and (*Lem. 2.*) $L : Am :: R : T, 64^{\circ}.02'$, the Direction sought.

C A S E XXXVIII.

A Piece whose horizontal Amplitude is AM , (*Fig. 19.*) = 7892, strikes an Object with the same Direction, whose Distance is $AX = 6520$, on a Plane whose Ascent is $MAX = 6^{\circ}.30'$, Quere that Direction.

Since

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Since $6^{\circ}.30'$ is one Angle of the Triangle AXM , $86^{\circ}.45' =$ half the Sum of the other two, and $AM + AX : AM - AX :: T, 86^{\circ}.45' : T, 59^{\circ}.11' =$ half the Difference of the other two Angles, whence $86^{\circ}.45' - 59^{\circ}.11' = 27^{\circ}.34' = AMX$, and (*per Theo. 13.*)

$T, AMX + T, MAX = T, 32^{\circ}.28'$, the Direction sought.

CASE XXXIX.

The Impetus Af (*Fig. 19.*) = 13068 Feet, and the Height of the Object $YX = 2217$, together with the Weight of the Shot = 63 Pounds given, to find the absolute Force at X .

$Af - YX = KX = 10851 =$ the Impetus at X , (*Theo. 10.*) and $10.4 \times 63 \times \sqrt{10851} = 68245$ the Force required in Pounds. (*Prop. 13.*)

CASE

C A S E XL.

The Direction $Y At$ (*Fig. 21.*) = $64^{\circ} 02'$, and Angle $Y Ax = 6^{\circ} 30'$, together with the absolute Force at the Point $x = 68245$ given, to find the Force wherewith the Plane Ax , or the Surface of any Object at x , of a given Inclination, may be struck by that Direction and Impetus.

Make dx parallel to AM , and (*Theo. 14.*) $T, Y At - 2 T, Y Ax = T, dxi$, and $dxi + Ax d =$ the Angle the Plane Ax makes with xi , the Tangent to the Point x , then (*Theo. 2.*) $R : S, Ax i :: 68245 : 63178$ the Force required.

Note, If the Surface of an Object at x , were in any given Position, by finding dxi , the Angle that the Object wou'd be struck at is found, and consequently, by the last Analogy, the Force required.

CASE

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C A S E XLI.

The Impetus Af (*Fig. 21.*) = 4000, and Height of an Object $Yx = 2000$, whose Surface reclines directly from the Perpendicular 32° , Quere the Direction and horizontal Distance whereat that Object may be struck with the greatest Force possible.

$Af - xY = 2000 = fx$ the Impetus at x , (*Theo. 10.*) then $R:S, 2 \mid \overline{32^\circ} :: 2000 : dx = 1798$, and because the Surface of the Object reclines 32° , the Angle $dx i$, which the Ordinate and Tangent to the Point x make, must be 32° , and (*per Theo. 9.*) $fxd = 90^\circ - 2 \mid \overline{dx i}$, therefore $R:\Sigma, 2 \mid \overline{32^\circ} :: 2000 : 877 = df$, and $dC(=xY) - df = Cf = 1123$, then $Af:Cf::R:S, fAC = 16^\circ. 18'$, and $45^\circ + \frac{1}{2}fAC = 53^\circ. 09'$ the Direction required: Lastly $R:S, AfC :: Af:AC = 3839$, and $AC + CY(=dx) = AY = 5637$ the Distance sought.

C H A P.

C H A P. III.

Of RANDONS made on Descending Planes, &c.

C A S E XLII.

THE greatest horizontal Randon of a Piece being 8000 = 2 Af, (Fig. 12.) Quere her greatest Randon on a Plane whose Descent is $YAX = 13^\circ$.

$R - S, YAX : R :: 2 Af : AX = 10322$ the Randon sought. (*Cor. 8. Theo. 12.*)

C A S E XLIII.

The gr. Ran. on a Plane whose Descent is 13° being $AX = 10322$,
Quere

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Quere her greatest Randon on the Plane of the Horizon.

$R : R - S, 13^\circ :: AX : 2 Af = 8000$
the Randon fought.

C A S E XLIV.

The greatest Randon on a Plane whose Descent is 13° being 10322, Quere the gr. Ran. on a Plane whose Descent is $6^\circ. 30'$.

$R - S, 6^\circ. 30' : R - S, 13^\circ :: 10322 : 9021$ the Ran. fought. (*Cor. 5 and 8. Theo. 12.*)

C A S E XLV.

The gr. Ran. on a Plane whose Descent is 13° being 10322, Quere the gr. Ran. on a Plane whose Ascent is $6^\circ. 30'$.

$R + S, 6^\circ. 30' : R - S, 13^\circ :: 10322 : 7186\frac{1}{2}$ the Ran. required. (*Cor. 5 and 8. Theo. 12.*)

Q

CASE

C A S E XLVI.

A Piece whose gr. Ran. on the Plane of the Horizon is 8000, being planted at *A*, (*Fig. 21.*) whose Height above the Plane of the Horizon is $AN = 2322$, Quere her gr. Ran. NX from that Height, and the Direction to reach it.

$8000 + AN = AX = 10322$. (*Cor. 7. Theo. 12.*) and (by 47. *El. 1.*) $\sqrt{AX^2 - AN^2} = NX = 10057\frac{1}{2}$ the Randon sought, then $AX : NX :: R : S$, $NAX = 76^\circ. 58'$, and $\frac{1}{2}NAX = 38^\circ. 29'$ the Direction sought. (*Cor. 2. Theo. 12.*)

C A S E XLVII.

A Piece whose Randon on the Plane of the Horizon at $64^\circ. 02' = MA$ (*Fig. 21.*) is $AM = 6060$, Quere her
Ran.

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Ran. with the same Elevation on a Plane whose Descent is $MAX = 13^\circ$.

$T, Mat + T, AXR = T, 66^\circ. 22' = T, RXM$, (*Cor. 1. Theo. 14.*) & $RXM - AXR = AXM = 53^\circ. 22'$, whence $AMX = 113^\circ. 38'$, and $S, AXM : S, AMX :: AM : AX = 6919$ the Randon fought.

CASE XLVIII.

A Piece whose Randon at $64^\circ. 02' = Mat$, (*Fig. 21.*) on a Plane whose Descent is $MAX = 13^\circ$, being $AX = 6919$, Quere her Ran. on the Plane of the Horizon at the same Elevation.

$T, Mat + T, MAX = T, 66^\circ. 22' = T, RXM$, (*Cor. 1. Theo. 14.*) and $RXM - AXR = AXM = 53^\circ. 22'$, whence $AMX = 113^\circ. 38'$, then $S, AMX : S, AXM :: AX : AM = 6060$, the Randon fought.

Q 2

CASE

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CASE XLIX.

A Piece whose Randon on the Plane of the Horizon at 40° . is 7580, Quere her Randon at $64^\circ. 02'$, on a Plane whose Descent is MAX (Fig. 21.) $= 13^\circ$.

$S, 2 \overline{40^\circ} : S, 2 \overline{64^\circ. 02'} :: 7580 : 6060 = AM$, (Cor. 6. Theo. 10.) then $T, 64^\circ. 02' + T, 13^\circ = T, 66^\circ. 22' = T, RXM$, and $RXM - RXA = AXM = 53^\circ. 22'$, whence $AMX = 113^\circ. 38'$, and $S, AXM : S, AMX :: AM : AX = 6919$ the Randon sought.

CASE L.

A Piece whose Randon at $38^\circ. 30' = MAI$, (Fig. 22.) on a Plane whose Descent is $MAD = 13^\circ$ being $AD = 1124$, Quere her Randon on the same Plane at $10^\circ. 35'$.

$T, MAI + T, MAD = T, 45^\circ. 44'$, whence the Angles at D and M are 32° .

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$32^{\circ}.44'$ and $134^{\circ}.16'$, (*Cor. 1. Theo. 14.*) then $S, AMD : S, ADM :: 1124 : 849 = AM$, the horizontal Ran. at $38^{\circ}.30'$, and (*per Cor. 6. Theo. 10.*) $S, 2 | 38^{\circ}.30' : S, 2 | 10^{\circ}.35' :: AM : Am = 315$, the Randon at $10^{\circ}.35'$, whence (*per Cor. 1. Theo. 14.*) the Angles at d and m , are $9^{\circ}.31'$ and $157^{\circ}.29'$, and $S, Adm : S, Am d :: Am : Ad = 730$ the Ran. required.

C A S E L I.

A Piece whose Ran. at $10^{\circ}.35'$ on a Plane whose Descent is $13^{\circ} = MAd$ (*Fig. 22.*) is $Ad = 730$, Quere her Ran. at $41^{\circ}.45'$ on a Plane whose Descent is $MAx = 6^{\circ}.30'$.

$T, 10^{\circ}.35' + T, 13^{\circ} = T, 22^{\circ}.31'$, whence the Angles at d and m are $9^{\circ}.31'$ and $157^{\circ}.29'$, (*Cor. 1. Th. 14.*) then $S, Amd : S, Adm :: Ad : Am = 315$, and (*per Cor. 6. Theo. 10.*) $S, 2 | 10^{\circ}.35' : S, 2 | 41^{\circ}.45' :: Am : AM = 866$ = the horizontal

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horizontal Randon at $41^{\circ}.45'$, then T ,
 $41^{\circ}.45' + T, 6^{\circ}.30' = \overline{T}, 45^{\circ}.11'$,
 whence the Angles at X and M are
 $38^{\circ}.41'$ and $134^{\circ}.49'$, lastly $S, 38^{\circ}.41' : S, 134^{\circ}.49' :: 866 : 983$ the
 Randon fought.

C A S E LII.

A Piece whose horizontal Ran. at
 $38^{\circ}.30'$ is AM (*Fig. 21.*) = 849, being
 planted at A with the same Elevation
 strikes an Object at X whose horizon-
 tal Distance is $NX = 1095$, Quere AN
 the perpendicular Height of the Piece.

$NX - AM = 246 = RN$, then
 $T, 38^{\circ}.30' : R :: AM : 1067 = L$,
 (*Lem. 2.*) and $L : NX :: RN : NA =$
 $252\frac{1}{2}$ fere, the Height required.

C A S E LIII.

A Piece planted at A (*Fig. 21.*) whose
 perpendicular Height above the Plane
 'of

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of the Horizon is $AN = 252\frac{1}{2}$, and there elevated $38^\circ 30'$, strikes an Object at X , whose horizontal Distance is $NX = 1095$, Quere her Ran. on the Plane of the Horizon with the same Elevation.

$NX : NA :: R : T$, $AXN = T, 13^\circ$,
and $T, 13^\circ + T, 38^\circ 30' = T, 45^\circ 44'$
 $= T, ARN$, (*Cor. 1. Theo. 14.*) then S ,
 $ARN : S, RAN :: AN : RN = 246$,
and $NX - RN = 849 = AM$, the
Randon required.

CASE LIV.

A Piece whose Randon on the Plane of the Horizon at $38^\circ 30'$ is $AM = 894$, (*Fig. 21.*) being planted at A whose Height above the Plane of the Horizon is $AN = 252\frac{1}{2}$, Quere how far she will carry from that Height with that Direction.

R:

$R:T, 38^{\circ}.30' :: \frac{1}{4} AM:CV = 177$,
 (Cor. 2. Theo. 9.) and $CV+AN=VP=$
 $429\frac{1}{2}$, then (Lem. 2.) $T, 38^{\circ}.30':R ::$
 $AM:L = 1067$, fere, and $\sqrt{L \times VP}$
 $= 677 = PX$, (Theo. 7.) lastly $\frac{1}{2} AM$
 $+ PX = NX = 1124$ the Distance re-
 quired.

C A S E LV.

A Piece whose Impetus is 4000, be-
 ing planted at A , (Fig. 5.) whose Height
 above the Plane of the Horizon is AP
 $= 624$, Quere how far she will carry
 with a horizontal Direction from that
 Height.

Thus $4 \times 4000 = 16000 = L$, (Cor.
 3. Theo. 7.) and $\sqrt{L \times AP} = 3160 =$
 $P K$ the Distance sought.

C A S E LVI.

A Piece whose gr. Ran. on the Plane
 of the Horizon is 8000, being planted
 at

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at *A*, (*Fig. 16.*) whose Height above the Plane of the Horizon is $AN = 188$, and there depress'd below the Horizon $7^{\circ}.30'$, Quere how far she will carry from that Height with that Direction.

$\frac{1}{2}$ gr. Ran. = 4000 = Af the Impetus, and (*Theo. 8.*) $Rf = Af + AN = 4188$, also, (by *Cor. 12. Theo. 12.*) $fAN = AfC$ = double the Depression, then $R : S, AfC :: Af : AC = 1036$, and $R : S, fAC :: Af : fC = 3864$, also $fC - AN = Pf$, and $\sqrt{Rf^2 - Pf^2} = PR$, lastly $PR - AC = RN = 970\frac{1}{2}$ the Distance required.

C A S E LVII.

A Piece planted at *A* (*Fig. 16.*) whose Height above the Plane of the Horizon is $AN = 188$, and there depress'd $7^{\circ}.30'$, lays her Shot at *R*, whose horizontal Distance is $RN = 970\frac{1}{2}$, Quere her Randon on the Plane of the Horizon at an Elevation of $38^{\circ}.30'$.

R

RN.

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$RN:AN::R:T$, $ARN=T$, $10^{\circ} 58'$, and (*Cor. 1. Theo. 14.*) $T, ARN-T$, $7^{\circ} 30' = T, 3^{\circ} 33'$, whence $\angle AN = 86^{\circ} 27'$, then $S, A \angle N: S, \angle AN:: AN: \angle N = 3049$, and $\angle N - RN = Am = 2078\frac{1}{2}$; lastly $S, 2 \overline{7^{\circ} 30'}: S, 2 \overline{38^{\circ} 30'}:: Am: 7825$ the Distance required.

CASE LVIII.

A Piece whose Impetus is Af (*Fig. 20.*) = 4000, Quere her Direction to strike an Object whose Distance is $AX = 7450$, on a Plane whose Descent is $YAX = 6^{\circ} 30'$.

$R: S, YAX:: AX: XY = 844$, and $Af + XY = fX$, (*Cor. 2. Theo. 8.*) then $AX: 2 Af + XY:: XY: X \angle = 1002$, and $\frac{AX - X \angle}{2} = AP = 3224$, also $Af: AP:: R: S$, $AfP = 53^{\circ} 42'$, and

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and $AfP + Pfd = Afd = 60^{\circ}.12'$,
 lastly $90^{\circ} - \frac{1}{2} Afd = 59^{\circ}.54' =$ the high-
 er, and $\frac{AfP - Pfd}{2} = 23^{\circ}.36' =$ the
 lower Elevation. (*Cor.* 12. *Theo.* 12.)

Note, If the higher Direction had been greater than the Complement of the Descent of the Plane, the lower wou'd have been a Depression, which is plain from the *Figure*.

CASE LIX.

A Piece whose Randon at $63^{\circ}.16'$, on a Plane whose Descent is MAX , (*Fig.* 21.) $= 6^{\circ}.30'$, is $AX = 7450$, Quere her Direction to strike an Object on the Plane of the Horizon whose Distance is 6000.

$T, 63^{\circ}.16' + T, 6^{\circ}.30' = T, 64^{\circ}.32'$,
 whence (*Cor.* 1. *Theo.* 14.) $AXM =$
 $64^{\circ}.32' - 6^{\circ}.30' = 58^{\circ}.02'$, and AMX
 $= 115^{\circ}.28'$; then $S, AMX : S, AXM$
 $R \quad 2 \qquad \qquad \qquad :: AX$

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$:: AX:AM=7000$, and $7000:6000$
 $:: S, 2 | 63^{\circ}.16'$; $S, 43^{\circ}.32'$, half of
 which is $21^{\circ}.46'$ the Direction sought.
 (Cor. 6. Theo. 10.)

CASE LX.

A Piece whose Impetus is Af , (Fig. 20.) = 4000, Quere her Direction to strike an Object at X , whose horizontal Distance is $AY=7402$, and whose Depth below the horizontal Line is $YX=844$.

$AY:YX::R:T$, $YAX=T, 6^{\circ}.30'$, and $S, YAX:R::YX:XA=7450$, then (per Cor. 12. Theo. 12.)
 $AX:2Af+XY::XY:XQ=1002$, and
 $\frac{AX-XQ}{2}=AP=3224$, also $Af:$
 $AP::R:S$, $AfP=53^{\circ}.42'$, and AfP
 $+Pfd=Afd$, lastly $90^{\circ}-\frac{1}{2}Afd=59^{\circ}.54'$ = the Direction required.

CASE

CASE LXI.

A Piece whose horizontal Amplitude is AM (*Fig. 21.*) = 7000, strikes an Object with the same Direction whose Distance is $AX=7450$, on a Plane whose Descent is $MAX=6^{\circ}.30'$, Quere that Direction.

Since $6^{\circ}.30'$ is the Angle included, $86^{\circ}.45'$ is half the Sum of the other two, and $AX+AM:AX-AM::T, 86^{\circ}.45':T, 28^{\circ}.43'$, whence $86^{\circ}.45'-28^{\circ}.43'=58^{\circ}.02'=AXM$, and $AXM+MAX=RXM=64^{\circ}.32'$, then (*Cor. 1. Theo. 14.*) $T, RXM-T, RXA=T, 63^{\circ}.16'$ the Direction sought.

CASE LXII.

A Piece planted at A (*Fig. 21.*) whose Height above the Plane of the Horizon

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zon is $AN = 844$, and elevated $20^\circ.14'$, strikes an Object at X whose horizontal Distance is $NX = 7402$, Quere her gr. Ran. on the Plane of the Horizon from that Height.

$NX : NA :: R : T, NXA = T, 6^\circ.30'$, and $T, NXA + T, 20^\circ.14' = T, 25^\circ.46' = T, ARN$; (*Cor. 1. Theo. 14.*) then $S, ARN : S, RAN :: AN : RN = 1748$, and $NX - RN = AM = 5654$, also $S, 2 | 20^\circ.14' : R :: AM : 8712 =$ the gr. Ran. (*Cor. 6 and 7. Theo. 10.*) and gr. Ran. + $AN = 9556 = AX$ (*Fig. 12.*) = the greatest Ran. on the Plane AX , (*Cor. 7. Th. 12.*) and $\sqrt{AX^2 - AN^2} = NX = 9518\frac{1}{2}$ the Randon sought.

CASE LXIII.

A Piece planted at A , (*Fig. 21.*) whose Height above the Plane of the Horizon is $AN = 844$, and there elevated $20^\circ.14'$ strikes an Object at X , whose horizontal

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zontal Distance is $NX = 7402$, Quere her greatest Ran. on a Plane whose Descent is 13° .

$NX : NA :: R : T$, $NXA = T$, $6^\circ 30'$, and $T, NXA + T, 20^\circ 14' = T, ARN$; (*Cor. 1. Th. 14.*) then $S, ARN : S, RAN :: AN : RN = 1748$, and $NX - RN = AM = 5654$, (*per Fig.*) and $S, 2 | 20^\circ 14' : R :: AM : 8712 =$ gr. Ran. on the Plane of the Horizon; (*Cor. 6 and 7. Th. 10.*) lastly (*Cor. 8. Th. 12.*) $R - S, 13^\circ : R :: 8712 : 11240$ the Randon required.

CASE LXIV.

A Piece planted at A , (*Fig. 21.*) whose Height above the Plane of the Horizon is $AN = 844$, and there elevated $20^\circ 14'$ strikes an Object at X , whose horizontal Distance is $NX = 7402$, being planted on the Plane of the Horizon, Quere how far she can strike an Object

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ject on the Top of a Perpendicular Y X
(Fig. 11.) = 886.

$NX:NA::R:T$, $NXA=T, 6^{\circ}.30'$; and $T, NXA+T, 20^{\circ}.14'=T$,
 ARN (Cor. 1. Theo. 14.) = $T, 25^{\circ}.46'$;
then $S, ARN:S, RAN::AN:RN$
= 1784, and $NX-RN=AM=5654$,
also (per Cor. 6 and 7. Theo. 10.) $S, 2$
 $[20^{\circ}.14':R::AM:8712 = \text{gr. Ran. \&}$
 $\text{gr. Ran.} - YX=7826$ (Fig. 11.) = AX ,
(Cor. 3. Th. 12.) lastly $\sqrt{AX^2 - YX^2}$
= $AY = 7775.7$ the Distance required.

C A S E LXV.

A Piece whose gr. Randon on the
Plane of the Horizon is 8000, Quere
her Direction and Distance, on a Plane
whose Descent is $6^{\circ}.30'$, to strike an
Object at X, (Fig. 21.) whose Surface
reclines directly from the Perpendicu-
lar $65^{\circ}.41'$, with the greatest Force
possible.

Since

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Since the Surface of the Object reclines $65^{\circ}.41'$, the Angle NXT , made by the Tangent to the Point X and the horizontal Line NX , must be $65^{\circ}.41'$, because the Stroke must be perpendicular; then $T, NX T - 2 T, AXN = T, 63^{\circ}.16'$, the Elevation, (*Theo.* 14.) and $T, 63^{\circ}.16' + T, AXN = T, NXM = T, 64^{\circ}.32'$, whence the Angles AXM and AMX are $58^{\circ}.02'$, and $115^{\circ}.28'$; then $R : S, 2 | \overline{63^{\circ}.16'} :: \text{gr. Ran.} : AM = 6428$, (*Cor.* 6. *Theo.* 10.) and $S, AXM : S, AMX :: AM : AX = 6841$ the Distance sought.

CASE LXVI.

The Impetus of a Piece being $Af = YK$ (*Fig.* 12.) = 4000, and the Weight of her Shot 63 Pounds, Quere the absolute Force at the Point X , whose Distance is $AX = 6841$, on a Plane whose Descent is $YAX = 6^{\circ}.30'$.

S

R:

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$R : S, YAX :: AX : XY = 774$,
and $XY + KY = 4774 =$ the Impetus
at the Point X (*Cor. 3. Theo. 10.*) in
Yards, which gives 14322 Feet, and
(*per Prob. 13.*) $10.4 \times 63 \times \sqrt{14322} =$
78410.8 the absolute Force in Pounds
required.

C A S E LXVII.

The Absolute Force at the Point X
(*Fig. 21.*) = 78410.8 and Direction
 $MA t = 63^\circ. 16'$, together with the
Descent of the Plane $MAX = 6^\circ. 30'$
being given, Quere the Force where-
with the Plane AX , or the Surface of
any Object of a given Inclination may
be struck at the Point X , by the same
Direction and Impetus.

$T, MA t + 2T, MAX = T, 65^\circ. 41' =$
 T, NXT (*Theo. 14.*) and $NXT - MAX$
 $= AX T = 59^\circ. 11' =$ the Angle the
Plane AX makes with XT , the Tan-
gent

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gent to the Point X ; then $R : S, 59^\circ$.
 $11' :: 78410.8 : 67340.8$ the Force re-
 quired.

Note, If the Surface of the Object to be struck were in any other given Position, by finding the Angle NXT , the Angle that it wou'd be struck at is found, and by the last Analogy the Force required.

C A S E LXVIII.

The greatest Impetus of a Piece being 4000, and the Requisite of Powder, for that Impetus, 27 Pounds, Quere the least Impetus and Requisite of Powder whereby that Piece can strike an Object at X , (*Fig. 20.*) whose Distance is $AX = 900$, on a Plane whose Descent is $MAX = 13^\circ$.

$$R : S, YAX :: AX : XY = 202, \text{ and}$$

$$\frac{AX - XY}{2} = 349 = \text{the least Impetus}$$

S 2
required;

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required; (*Cor. 7. Theo. 12.*) then
 $\sqrt{4000} : \sqrt{349} :: 27 : 8$ nearly, the
 Charge required. (*Prob. 11.*)

CASE LXIX.

A Piece whose Impetus with 27 Pounds of Powder is 4000, being planted at *A* (*Fig. 21.*) whose Height above the Plane of the Horizon is $AN = 202$, Quere her Direction and least Charge of Powder that can reach an Object at *X*, whose horizontal Distance is $NX = 877$.

$AX : NX :: R : T$, $NAX = T, 77^\circ$, half of which is $38^\circ. 30'$ the Elevation required, (*Cor. 2. Tb. 12.*) and $S, NAX : R :: NX : AX = 900$, also $\frac{AX - AN}{2} = 349$ the least Impetus; (*Cor. 7. Theo. 12.*) then $\sqrt{4000} : \sqrt{349} :: 27 : 8$ the Charge required,

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C A S E LXX.

A Piece whose Impetus with 27 Pounds of Powder is 4000, Quere her least Impetus and Requisite of Powder to strike an Object at X , (*Fig. 19.*) whose Distance is $AX = 1600$, on a Plane whose Ascent is $YAX = 6^\circ. 30'$.

$R : S, YAX :: AX : XY = 182$,
and $\frac{AX + XY}{2} = 891 =$ the least Im-
petus ; (*Cor. 3. Theo. 12.*) then $\sqrt{4000} :$
 $\sqrt{891} :: 27 : 12\frac{3}{4}$ the Charge required.

C A S E LXXI.

A Piece whose Impetus with 27 Pounds of Powder is 4000, Quere her Direction and least Charge of Powder that can strike an Object at X , (*Fig. 19.*) whose Height above the Plane of
the

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the Horizon is $YX = 182$, and whose horizontal Distance is $AY = 1590$.

$AY : XY :: R : T, YAX = T, 6^\circ. 30'$, and $45^\circ + \frac{1}{2}YAX = 48^\circ. 15' =$ the Direction ; (*Cor. 2. Theo. 12.*) then $S, YAX : R :: YX : AX = 1600$, and $\frac{AX + XY}{2} = 891 =$ the least Impetus, (*Cor. 3. Theo. 12.*) also $\sqrt[th]{4000} : \sqrt[th]{891} :: 27 : 12\frac{3}{4}$ the Charge required.

SCHOLIUM IV.

These Rules for finding the Direction and adjusting the Requisites of Gun-Powder to strike an Object with the least Impetus possible, are very useful for saving Ammunition, especially in throwing of Bums, which are not so often design'd to do Damage by their projectile Force as by the breaking of their Shells, setting Houses on Fire, and the like ; but because these Effects are

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are often prevented by not proportioning the Fuse to the Time of the Flight, it will be proper to give some Rules for computing those Times. Now, (by *Cor. 2. Theo. 7.*) the Time of the Flight of any Projection made on the Plane of the Horizon, is double the Time of the Fall of a heavy Body from the Height of that Projection; and the Times of the Falls from different Heights, being (by *Cor. 2. Theo. 5.*) in the subduplicate Ratio of those Heights, and also the Fall in one Second of Time, being, in round Numbers, 16 Feet, if h = the Height of any Projection made on the Plane of the Horizon, it will be

$$\sqrt{16} : 1'' :: \sqrt{h} : \frac{\sqrt{h}}{4}, \text{ equal the Time of}$$

the Fall from the Height of the Projection, and $\frac{1}{2} \sqrt{h}$ = the Time of the Flight of the Shot.

CASE

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CASE LXXII.

The greatest Randon of a Piece being AM , (*Fig. 13.*) = 25600 Feet, Quere the Time of the Flight of her Shot.

$\frac{1}{2}AM = 6400 = Fv$ the Height of the Projection; because $Fv = vT = \frac{1}{2}AF$, (*Cor. 1 and 2. Theo. 9.*) and (*per Scholium 4.*) $\frac{1}{2}\sqrt{6400} = 40''$ the Time required in second Minutes.

Note, If the Time of the Flight at any Elevation be counted by Vibrations of a Pendulum of any convenient Length, the Time of the Flight at any other Elevation is had. (*Theo. 11.*)

CASE LXXIII.

The Time of the Flight of a Shot made at 45° , on the Plane of the Horizon being 40 Vibrations, Quere the Time at $7^\circ. 13'$.

$S, 45^\circ.$

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$S, 45^\circ : S, 7^\circ. 13' :: 40 : 7$ the Vibrations required.

C A S E LXXIV.

A Piece whose Randon at $52^\circ. 30'$ is $AM = 4356$, (*Fig. 19.*) the Time of the Flight of her Shot being 24 Vibrations, Quere the Time of its Flight to X , whose horizontal Distance is $AY = 3446$.

$Am : AY :: 24 : 19$ the Time required.

C A S E LXXV.

A Piece whose horizontal Randon at $52^\circ. 30'$ is Am (*Fig. 16.*) = 4356, the Time of the Flight of her Shot being 24 Vibrations, being planted at A , above the Plane of the Horizon, with the same Direction placeth her Shot at Q , whose horizontal Distance from

T
the

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the Perpendicular AN is $NQ = AM = 5808$, Quere the Time of the Flight to Q .

$Am : AM :: 24 : 32$ the Time required. (*Cor. 3. Theo. 6.*)

CASE LXXVI.

A Piece whose horizontal Randon at $52^\circ. 30' = m AL$, is Am (*Fig. 16.*) = 4356, the Time of the Flight of her Shot being 24 Vibrations; the same Piece being planted at A , (*Fig. 5.*) with a horizontal Direction AI , placeth her Shot at K , below the horizontal Line, whose horizontal Distance is $PK = AI = 4198$, Quere the Time of the Flight from A to Q .

$90^\circ. - m AL$ (*Fig. 16.*) = $AL m = 37^\circ. 30'$, then $S, 37^\circ. 30' : R :: Am : AL = 7156$, and (*per Cor. 4. Theo. 6.*) AL (*Fig. 16.*) : AI (*Fig. 5.*) :: 24 : 14 the Time required.

CASE

CASE LXXVII.

A Piece whose horizontal Randon at $7^{\circ}. 13'$ is 1086, the Time of the Flight of her Shot being 4 Vibrations; the same Piece being planted at *A*, (*Fig. 16.*) above the Plane of the Horizon, and there depress'd $7^{\circ}. 13'$ placeth her Shot at *R*, whose horizontal Distance from the Perpendicular is $RN = 814$, Quere the Time of the Flight of her Shot from *A* to *R*.

$1086 : 814 :: 4 : 3$ the Time required.

P A R T. III.

The Description and Use of a new Mathematical Instrument.

HAVING in the foregoing Pages given the Demonstration and Solution, by Calculation, of all the Cases, I think requisite in the Projectile Part of Gunnery, I shall now proceed to give the Description and Use of an Instrument, which, by placing its Parts in certain Positions, will solve any Problem before mentioned by Inspection only; as also, with a small Addition, measure the Height, or Distance of an Object on any Plane accessible, or inaccessible, together with the Ascents, or Descents of those Planes.

Tho' Instrumental Solutions are not so exact as those done by Calculation, if this Instrument is accurately made, the Ease and Expedition of it will more than recompence a small Fraction of an Error, especially to the Practical Ingeneer, who, in the Heat of Action, may often have Occasion for new Directions, and several other Things which he has not leisure to find by Calculation.

Ler

The Description of, &c. 141

Let a Brass Semicircle (*Fig. 26.*) of about nine Inches Radius, have its inner Edge, or Limb, divided into 90 equal Parts, beginning at *N* and counting upwards 10, 20, 30, &c. to 90, at *Z*, and each of those Divisions subdivided into 6 equal Parts. Let the outer Limb be divided into Degrees and 6th. Parts of a Degree, marking the Degrees from the Middle of the Limb both Ways 10, 20, 30, &c. to 90 at *N* and *Z*. Let also the middle Space, between the outer and inner Limbs, be mark'd from *Z* to *N*, 10, 20, 30, 40, &c. to 180 at *N*.

Let this Semicircle be fixt to the Middle of a Box Ruler *BD*, about $3\frac{1}{2}$ Foot long, an Inch and half broad, and of a convenient Thickness. The inner Half of the Breadth of this Ruler must be level with the Surface of the Semicircle, but the outer Half must be higher about $\frac{2}{3}$ of an Inch. On the
outer

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outer Half there must be fixt a thin Brass Scale of an equal Length and Breadth with the Box Ruler, the Breadth of which Scale is to be divided, by Lines drawn from End to End, into three equal Parts, and the Length into Inches, half Inches, and 10th. of an Inch; the Inches are to be drawn directly cross the whole Breadth, and mark'd 1. 2. 3. 4. &c. both Ways to *B* and *D*; the half Inches are to be drawn cross the middle and innermost Third, and the 10ths. only cross the inner Third. Let there be on one End of this Scale an Inch, and on the other End half an Inch, each divided very exactly into ten equal Parts Diagonal Ways, that the Tenths and Centesms which may happen in the Operations, on the Square and Indices hereafter to be describ'd, may be exactly measur'd on them by a Pair of Compasses. The Reason for raising the outer Half of the Box Ruler, above the inner Half, two Tenths of an Inch, as before

an Instrument, &c. 143

before mentioned, is to make room for the Indices *Ab* and *Ad*, which are to be fixt to the Center of the Semicircle, and there to open and shut, as Occasion requires, like the Legs of a Sector. Those Indices are to be about 26 Inches long, $\frac{3}{4}$ of an Inch broad, and about $\frac{1}{10}$ thick; their Breadth is to be divided into three equal Parts, and their Length into Inches, half Inches and Tenths, as the Brass Scale before mention'd: the Inches are to be mark'd from the Center *A*, 1. 2. 3. 4. &c. to *b* and *d*, and the Tenths drawn cross the inner Third. Each of those Indices must have a small Screw Nut with a Pin, or Bit of Wire upon it, which Pin may, by the Screw Nut, be fixt exactly to any Division on them in order to suspend the Label, or Ruler *TY*, which has a thin Piece of Brass with a small Hole in it, exactly fitting the foresaid Pin, and is to be fixt also to any Division of the Ruler as Occasion requires. Let this Label,

or

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or Ruler, be about two Foot long, and of the same Breadth and Thickness with the Indices *Ab* and *Ad*, and divided after the same Manner as they are, only the Tenths are to be drawn cross the inner Edge as well as cross the inner Third of the Breadth, and the Inches are to be mark'd 1. 2. 3. 4. &c. from *C* to *T*, and *Y*, making *CT* eighteen Inches, and *CY* six. The like Divisions are to be made on the Side of the Square *KX*, beginning at the inner Edge of the Brass Ruler at *K*, marking the full Inches on the upper Side 1. 2. 3. 4. &c. to 24, the Tenths are to be drawn cross the upper Third and the upper Edge. Let this Instrument be fixt upon Feet with a Ball and Socket like those of a common surveying Instrument, but stronger in order to keep it very firm; and let there be Sights which may, as Occasion require, be fixt on the Diameter, Indices and Ruler *TY*, of the same Kind with those of the Surveying

an Instrument, &c. 145

Surveying Instrument before mentioned, which are too well known to want a farther Description.

Note, the Ball and Socket must not be fixt exactly under the Center of the Semicircle, but some Distance from it, on the Cross-Bar which goes from the Center to the Middle of the Limb, as well to support the Head of the Instrument more easily, by being nearer its Center of Gravity, as to make room for an Air-level made with a Glass Tube and Spirits of Wine, which must be fixt exactly under the Diameter, or Ruler *AB*, so that when the Semicircle is turn'd vertically the Diameter may be fixt in a horizontal Position.

The Instrument being thus prepar'd let there be a Chain of 100 Links marked with double Rings at each 10 Links, in order to have the Fractions which may happen in measuring any Distance in Tenths and Centesms, agreeing with

U the

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the Diagonal Scale before mentioned : this Chain may be of any convenient Length, but because most of the new *English* Pieces of Ordnance have their greatest horizontal Randons known, or engraven on them in Paces, or Yards, in my Oppinion, 100 Foot will be a convenient Length, for if every Chain be reckon'd ten Integers if the gr. Ran. of any Piece is multiply'd by 3, and one Figure cut off for a Decimal, you will have that Randon in Tenths of your Chain, or in Integers, each of which is ten Foot.

E. g. The gr. Ran. of a Cannon Royal is, commonly, computed to be 8000 Yards, this multiply'd by 3 and a Cypher cut off ; gives 2400 now two Foot, or 24 Inches on the Index of your Instrument, being taken for this 2400, $\frac{1}{100}$ of an Inch, which is very distinguishable on the Diagonal Scale, is 10 Foot on the Ground, or an Integer

an Instrument, &c. 147

ger of the foresaid 2400. The Ingenious Practitioner may find out some useful Alterations in both the Length and Division of the Parts of the Instrument and Chain ; to such I leave the farther Consideration of it ; and shall next proceed to

T H E

USE of *the* INSTRUMENT
in LONGIMETRY, or mea-
suring HEIGHTS and DIS-
TANCES.

E X A M P L E I.

LET it be required to find the Distance from the House at *A*, to the Castle, (*Fig. 24.*) or to any Part thereof as the Weathercock on the Top of the Spire at *C*.

Having set up your Instrument at *A*, turn it about till through the Sights on

U 2 the

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the Diameter you spy a Mark set up at *B*, in any convenient Place of the Field about you, and having fix'd the Diameter in that Position, turn the moving Index till through the narrow Slit of a small Sight fix'd upon the Center you see the Hair in the other Sight cut the Spire at *C*, then fixing the Index in that Position to the Limb of the Semi-circle, measure with your Chain in a straight Line from *A* to *B*; and having mark'd the Chains and Links of that Distance on the Diameter, and plac'd the Ruler with the Sights on it exactly to that Distance by means of the small Pin and Hole mention'd before, set up your Instrument just at the End of the Distance you measur'd, (which, if you please, you may take all in full Chains, if the Ground will allow it, for you need not care whether you are short of the Mark *B*, or over it, provided you keep in the same straight Line) and turn it about till through the Sights on the Diameter
ameter

ameter you spy a Mark set up at your first Station *A*, and having fix'd it in that Position, turn the Ruler on the Pin, which is fix'd at the former Distance on the Diameter, till through the Sights on it you spy the Weathercock at *C*, then will the Part of the Index *ac*, cut by the inner Edge of the Ruler, give the Distance *AC* from the House to the Spire at *C*, which was to be found; and if there is Occasion the Distance from the Mark at *B*, to the Spire will be found on the Ruler at the Intersection of the Index; all which is plain from the Similarity of the Triangles *ABC*, and *a, Pin, c*, or that form'd by the Diameter, Index and Ruler, and *Cor. 1. 4. El. 6.*

Note, If the Distance from *A*, to two, three, four, &c. Objects, had been required, if when the Diameter was first fix'd in the Position *AB*, the Index had been turn'd to each of those Objects
and

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and the Degrees cut by the Index on
the Limb at the Direction to each re-
spective Object noted down in a little
Field Book. Thus,

Degrees.	Distances.	Objects.
34 $\frac{1}{2}$		Wind Miln.
10		Tree,
25 $\frac{1}{2}$		Bust on the Wall.
40 $\frac{3}{4}$		Weathercock.

Then the Distance *AB* being mea-
sur'd, and the Instrument fix'd at *B*, as
before directed, if the Index is plac'd
successively to the Degrees noted down
in the Field Book, the Ruler directed
to each respective Object will cut upon
the Index the Distance of that Object
from the House at *A*, which may be set
down for farther Use, in the Distance
Column of the Field Book; and if there
is Occasion for the Distance from *B*,

to

to each Object, or for the Distances of those from each other, they may be easily found; for when you direct the Ruler to any Object the Index being at the Degrees, or Angle belonging to that Object, will cut the Distance from it to *B*, on the Ruler, and if one of the Indices be fix'd to the Degrees belonging to one Object, and the other Index to those belonging to another Object, the Distance between the two Points on those Indices which represent the Distances of the respective Objects from *A*, will give the Distance of those Objects from each other, which may be either taken in a Pair of Compasses and apply'd to a Scale, or found by applying the Beginning of the Divisions on the Ruler to one of those Points, and the other Point will cut the Distance between the Objects upon it; but from the Angles and first Distances found in the Field, the rest may be found by the
In-

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Instrument at any Time or Place you have Occasion for them.

II.

Let it be required to find the Perpendicular Height YX (*Fig. 25.*) of the Top of the Wind-Miln above the horizontal Line HO , as also the horizontal Distance AY , of that Perpendicular.

Find the Distance AX , by the last *Case*, and having plac'd the Semicircle vertically in the Plane AXY , and with its Diameter in the horizontal Line HO , (which last may be done by a Line and Plummet, or by the Air-level before mentioned) move the Square along the Diameter until it cuts the first found Distance on the Index, then will the Index cut the Height required on the Square, and the Square the Horizontal Distance on the Diameter, which was to be found.

Note,

Note, If there is any Occasion for the Angle of Ascent YAX , the Index cuts it on the outer Limb; and if the Height of the Wind-Miln from the Foundation is required, let the Semicircle continue in the same Position, and move the Index till through its Sights you spy the Foundation at f , and the Difference of what it now cuts on the Square and what it cut before is the Height required. It may not be improper to remark here, that this Part of the Proposition relating to the Height of the Wind-Miln wou'd require that the Distance from it to the Instrument shou'd be but small, that it may be plainly express'd on the Index in Feet, which will, at least, require that every Chain of the Distance AX , be an Inch on the Index; now, the Index being but 26 Inches long, it's plain the Distance AX , ought not to exceed 26 Chains, or 2600 Feet.

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III.

How to measure the Depth SH , of the Ship (*Fig. 25.*) at the Surface of the Water below the horizontal Level of the Instrument at B .

This Proposition is perform'd after the same Manner the last was, only that it will be most convenient to turn the Limb of the Semicircle downwards as it appears in the *Figure* at B .

These three Examples contain all that the practical Ingeneer has Occasion to know relating to the Mensuration of Heights and Distances, whether they are accessible, or inaccessible ; for in either Case there is nothing more required but that he may see the Object from two convenient Stations sufficiently distant from each other, on any Plane whether ascending, descending, or horizontal.

T H E

THE

USE of the INSTRUMENT
in PRACTICAL GUNNERY.

IN what follows I will not trouble the Reader with Quotations, or References to former Propositions, because the Thing is so plain and easy, and so naturally follows from what is said before, that none who tolerably understands that, can be at a loss for the Reasons of this.

And the principal Propositions a practical Ingeneer has Occasion for, being, from any two of the Impetus, Distance, and Direction given, how to find the Third; I shall give Examples in all the possible Cases of these, which will sufficiently shew the Ease and Expedition of this Instrument, and lead the apt Practitioner to the Use of it in other Propositions when Occasion requires.

Note, Whenever the moving, or placing, of the Square is mentioned, it is suppos'd to slide along the Diameter, or Ruler, still keeping K X at Right-Angles, or Perpendicular to BD.

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P R O P. I.

The greatest horizontal Randon of a Piece being 2400, Quere her Direction to strike an Object whose horizontal Distance is 1584.

Place the Distance on the Square to the gr. Ran. on the Index, and the latter will cut $69\frac{1}{2}$ Degrees on the inner Limb of the Semicircle, which is the Direction required.

P R O P. II.

A Piece whose gr. Ran. on the Plane of the Horizon is 2400, Quere her Randon at $69\frac{1}{2}$ Degrees.

Fix the Index to the Direction on the inner Limb, and the greatest Randon on it will cut 1584, on the Square the Randon sought.

P R O P.

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P R O P. III.

A Piece whose horizontal Randon at $69\frac{1}{2}$ Degrees is 1584, Quere her greatest horizontal Randon.

Fix the Index to the Direction on the inner Limb, and the given Randon on the Square will cut on the Index 2400 the gr. Ran. required.

These are the principal Propositions relating to Projections made on the Plane of the Horizon: But if the Height and Sublimity of the Projection were required, the Square in the last Position will cut 1803 on the Diameter, one fourth the Sum and Difference of which, and the greatest Randon is 1051, and 149, the Height and Sublimity required.

P R O P. IV.

The Impetus of a Piece being 1200, Quere her Direction to strike an Object

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ject whose Distance is 1728, on a Plane
whose Ascent is $6\frac{1}{2}$ Degrees.

Fix the lower Index *Ad*, (*Fig. 26.*)
to the Ascent of the Plane on the outer
Limb of the Semicircle, then move the
Square to the Distance of the Object
upon it, and mark the Distance *AK* cut
on the Diameter by the Square; then
having suspended the Ruler at *f*, by
means of the small Pin and Hole before
described, making *Af* and *fY* each
equal to the given Impetus, move the
Ruler *fY*, exactly along the given Di-
stance of the Object upon the Index at
X, until *XY* is equal to *AK*, then
will the Index *Ab*, cut $65\frac{1}{4}$ Degrees, on
the inner Limb of the Semicircle the
Direction required.

P R O P. V.

A Piece whose Impetus is 1200,
Quere her Randon at $65\frac{1}{4}$ Degrees, on
a Plane whose Ascent is $6\frac{1}{2}$ Degrees.

Fix

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Fix the lower Index to the Ascent of the Plane on the outer Limb, and the other to the Direction on the inner Limb, making Af , and fY , (*Fig. 26.*) each equal to the given Impetus,

Lastly, Move the Square and Ruler (keeping their Intersection exactly to the inner Edge of the Index Ad) until AK , and XY are exactly equal, then will $AX = 1728$ the Randon required.

P R O P. VI.

A Piece whose Randon at $65\frac{1}{2}$ Degrees is 1728, on a Plane whose Ascent is $6\frac{1}{2}$ Degrees, Quere the Impetus of that Piece.

Fix the lower Index as *per* last Case, and the Ruler to the given Distance at X , by means of the Pin and Hole, making XY equal to AK ; lastly turn the Ruler on the Center X , and the Index Ab , on the Center A , until Af and fY are

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are equal, and each of them will be
1200 the Impetus required.

P R O P. VII.

A Piece whose greatest Randon on
the Plane of the Horizon is 2400,
Quere her greatest Randon on a Plane
whose Ascent is $6\frac{1}{2}$ Degrees.

Fix the lower Index to the Elevation
of the Plane, and move the Square a-
long the Diameter until $AK + AX$ be
equal the greatest Randon given, then
will AX be 2156 the gr. Ran. sought.

Or thus,

Place the Square to 1000, on the
Index, and it will cut 113 on the Dia-
meter, then place $1000 + 113$ on the In-
dex to 1000 on the Square, and having
fix'd the Index, move the Square to
2400, the given Randon on the Index,
and it will cut 2156 on the Square the
greatest Randon sought. *PROP.*

P R O P. VIII.

A Piece whose greatest horizontal Randon is 2400, Quere the greatest horizontal Distance from which that Piece can strike an Object on the Top of a Perpendicular whose Height is 400.

Place the Square to 400, on the Diameter; and 2000 on the Index (the Complement of the Height to the given gr. Ran.) will cut 1685 on the Square which is the horizontal Distance required.

P R O P. IX.

A Piece whose gr. Ran. on the Plane of the Horizon is 2400, Quere the greatest Height which that Piece can reach on a Perpendicular whose horizontal Distance is 1685.

Y

Keep

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Keep the Edge of the Index to the given horizontal Distance on the Square, and move the Latter until the Sum of the Numbers cut on the Diameter and Index be equal to 2400 the given gr. Ran. and those on the Diameter will be 400 the Height required.

P R O P. X.

A Piece whose Impetus is 1200, Quere, her Direction to strike an Object whose Distance is 2235, on a Plane whose Descent is $6\frac{1}{2}$ Degrees.

Fix the lower Index to the Descent of the Plane, and the Ruler to the given Distance upon it at *X*, (*Fig. 27.*) making $CX = AK$ on the Diameter; lastly, make *Af*, and *fC*, each equal to the given Impetus, and the Index *Ab*, will cut on the inner Limb $59\frac{1}{2}$ Degrees the Direction sought.

P R O P.

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P R O P. XI.

A Piece whose Impetus is 1200,
Quere her Randon at $59\frac{1}{2}$ Degrees, on
a Plane whose Descent is $6\frac{1}{2}$ Degrees.

Fix the lower Index to the Descent
of the Plane, and the upper Index to
the Direction on the inner Limb of the
Semicircle, and having suspend the Ru-
ler at *f*, making *Af*, and *fC*, each equal
to the Impetus, move the Square and
Ruler, (keeping their Intersection ex-
actly to the inner Edge of the lower In-
dex) until *AK*, and *CX* are equal, then
will *AX* be 2235 the Randon sought.

P R O P. XII.

A Piece whose Randon at $59\frac{1}{2}$ De-
grees is 2235, on a Plane whose Descent
is $6\frac{1}{2}$ Degrees, Quere the Impetus of
that Piece.

Y 2

Fix

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Fix the Indexes as *per* last *Case*, and the Ruler to the given Distance on the lower Index making CX equal to AK , on the Diameter ; lastly, turn the Ruler and upper Index until Af , and fC are equal, and each of them will be 1200 the Impetus required,

P R O P. XIII.

A Piece whose Impetus is 1200, Quere her gr. Ran. on a Plane whose Descent is $6\frac{1}{2}$ Degrees.

Fix the lower Index to the Descent of the Plane on the outer Limb, and the Square mov'd to 1000 upon it will cut 113, on the Diameter, the place $1000 - 113 (= 887)$ on the Square to 1000, on the Index, and having fix'd the Latter, move the Square until the Index cuts 1200, the given Impetus upon it, then will the Square cut 1353, on the Index double, which is 2706 the gr. Ran. required.

P R O P.

P R O P. XIV.

A Piece whose greatest horizontal Randon is 2400, being planted upon an Eminence whose perpendicular Height above the Plane of the Horizon is 306, Quere her gr. Ran. from that Height.

Fix the Square to the given Height on the Diameter, and 2706 (the Sum of the given Height and gr. Ran.) on the Index will cut on the Square 2686 the Randon required.

Note, The Sum of the given Numbers, and the Number sought are each too great to have the first two Figures in Tenths of an Inch on the Instrument they may therefore be taken in Tenths of a half Inch ; or if the Halves of the given Numbers are taken on Inches as before, half the Number sought will be got on the Index.

P R O P.

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P R O P. XV.

The horizontal Distance and Height or Depth of an Object, either above or below the horizontal Level of the Piece, being given to find the least Impetus that possibly can reach that Object.

Fix the Square to the given Height or Depth on the Diameter, and move the Index to the horizontal Distance on the Square; then will half the Sum of the given Height and Distance cut on the Index in Ascents, and half their Difference in Descents, be the Impetus requir'd.

S C H O L I U M.

It is plainly demonstrated, by *Theo.* 12. that the Direction to reach an Object with the least Force possible bisects the
the

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the Angle between the Object and Zenith, and, by the 2d. and 7th. *Corollaries* to that *Theorem*, that Force is such as wou'd carry the Shot on the Plane of the Horizon, at an Elevation of 45 Degrees, a Distance equal to half the Sum of the perpendicular Height and Distance of the Object in Ascents; and half the Difference of the perpendicular Depth and Distance in Descents; all which are found with the greatest Ease imaginable by the Examples of the Use of the Instrument in *Longimetry*.

Now having that Direction and Force, together with the Force or Impetus of his Piece with any given Charge, which is found by the 6th. or 12th. *Prop.* foregoing Propositions, the Engineer has nothing to do with Regard to hitting an Object, but to adjust his Charge of Powder by *Case 11. Page 88.* and direct his Piece according to the given Elevation, which he may do
by

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by the Method describ'd in *Page 79.* or by Dr. *Halley's* Method of fixing a Piece of Looking-glass Plate parallel to the Muzzle of the Mortar, which is suppos'd to be cut square to the Bore, and to raise or depress the Elevation, till the Object appears reflected from the same Point of the Glass, where on a Lead Plumbet, hung from the Observers Eye, falls.

I do not pretend to direct Engineers in Things that are only practical, but, as Care and Diligence are as necessary as Knowledge in every Employment, I hope they will excuse me for telling them that, whether they are the Besiegers, or the Besieged, the Distances, Directions, and Requisites of Powder for reaching any particular Object, or Pass, which their General may think proper to attack, or defend, ought all to be ready calculated beforehand, when the Nature of the thing will allow of it.

SOME

S O M E

New PROPERTIES of PROJECTILE CURVES.

THE following Properties are not of any material Use in the Art of Gunnery, but as their Demonstrations depend on the foregoing Theory, I thought it not amiss to insert them here: For, beside the Satisfaction they may probably give the curious Readers, there may perhaps be some latent Use in them which time and their farther Consideration may discover.

The 4th. Property is hinted at by the learned Dr. Halley who shews that the utmost Limits of the Reach of every Project, made by the same Impetus, is in the Curve of a Parabola, &c. but he does not there observe, that a Line drawn from the Engine to any Point in those Limits shall pass through the Focus Point of the respective Parabola whose utmost Limit that Point is, which leads to a very plain and easy Method to find the Direction necessary

Z

to

to hit any Object within the Reach of any given Impetus, whether that Object be below or above the horizontal Level of the Piece.

The rest of these Properties together with those discovered by Theo. 13 and 14, and Corollary 4th. to Theo. 10th. I have never seen or heard of before, nor am I any way vain of the Discovery, tho' I insert them here as new, for he is but a small Proficient in Mathematicks that could not discover new Properties in Curves every Day of his Life, if he employ'd his Mind that Way; yet so extensive is Science, and so limited human Capacity, that the simplest Curve wou'd afford Employment to the greatest Genius that ever was, tho' he were to live a thousand Years.

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I.

THE Points of Intersection of the Tangent and Axis produc'd, of every Parabola that can be describ'd by the same Impetus in the same vertical Plane, will be in the Circumference of a Circle whose Radius is the Impetus, and whose Center the highest Point of that Impetus.

By *Cor. 1. Theo. 9.* the distance fT (*Fig. 9.*) from the Focus to the Intersection of the Tangent and Axis produc'd, is still equal to the Impetus Gf , or GR ; and since the End f (*Fig. 13.*) of the Line ft is still in the Circumference of a Circle, (*Cor. 4. Theo. 10.*) ft , being still equal and parallel to it self, the End t must also be in the Circumference of a Circle, of which it's Evident Z must be the Center.

II.

The principal Vertexes $V.v.u.$ &c. (*Fig. 13.*) of every Parabola describ'd by the same Impetus, &c. is in the Periphery of an Ellipsis whose conjugate Diameter is the Impetus AZ , and whose transverse Diameter is double that Impetus.

By *Cor. 7. Theo. 10.* 'tis evident the principal Vertex v of the Parabola AvM describ'd

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scrib'd by an Elevation of 45 Degrees is at the greatest Distance possible from the Impetus AZ , that is pv equal and parallel to AF , = AZ ; and if the Direction was in the same Plane on the other Side, or depress'd on this Side 45 Degrees, the principal Vertex wou'd be in vp produc'd that Way as far distant from the Impetus AZ on that Side as it is on this; consequently $2pv = 2AZ$ is the longest Axis of the Curve. Now because $Fv = vT$, (*Cor. 2. Theo. 9.*) and $VK = Cu$, (*Cor. 9. Theo. 10.*) the Line vp , produc'd when the Vertexes fall on the other Side, will still bisect the Line Vu that joyns the Vertexes of any two Parabola's made by Elevations equally above and below 45 Degrees. Those Vertexes are therefore in the Periphery of an Ellipsis. Q. E. D.

Or thus,

Let $Af = AZ = x$, and $AC = pn = y$, (n being taken any where in the right Line pv) then will $\sqrt{xx - yy} = fC$. But because $fV = Cu$, and pn bisects nV from above, $\frac{\sqrt{xx - yy}}{2} = Vn$, the Ordinate to the Point

n ; which Equation is expressive of an Ellipsis; for, in that Figure the Square of the Transverse Diameter, is to the Square of the Conjugate, as the Rectangle of any two Abscissa's, to the Square of the Ordinate; that is,

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is, by the Substitution and what was said a-

bove, $4xx : xx :: xx - yy : \frac{xx - yy}{4}$

which is evidently the Square of $\sqrt{\frac{xx - yy}{2}}$

the Value of nV found before. Q. E. D.

III.

If C (*Fig. 28.*) is the Place of an Engine, and CZ its Impetus, all the Parabola's that can be describ'd by that Impetus, &c. will have the outward Extremities of their Parameters, or that pointing to the Right-hand, in the Periphery of an Ellipsis $ZqMC$, whose Area is equal to the Area of the Circle $ZfNS$, described on the Center C with the Radius CZ .

Let ZM be the Diagonal of a Square describ'd on the Diameter ZN , equal to twice the Impetus, and let an indefinite Number of right Lines $gr. dn. Sq.$ &c. be suppos'd to be drawn parallel to ZB or NM . By *Cor. 4. to Theo. 10.* all the Parabola's that can be describ'd as above, will have their Focus Points in the Circumference of the Circle $ZfNS$, &c. and by *Theo. 7.* the Semi-parameters $pr. mn. fq.$ &c. are still equal to the correspondent Heights $Za. ZK. ZC.$ &c. or their Equals $ab. Kl. Cf.$ &c. that is $ab =$
 $pr.$

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pr. $Kl = mn$. $Cf = fq$. $eo = XV$, &c. hence, by the Doctrine of Indivisibles, the Triangles Zab . ZKl . ZCf . &c. are still equal to the corresponding mixt lin'd Figures Zpr . Zmn . Zfq . &c. and the whole mixt lin'd Figure $ZqVMNf$ equal to the Triangle ZNM , or half the Square of the Circles Diameter. Now since the Line of Direction still bisects the Angle between the Focus and Zenith, (*Theo. 9.*) if the Direction is horizontal the Focus Point must be in the Nadir at N . But if the Direction be depressed below the Horizon the Focus will be some where in the Semi-circumference NSZ : But still the Semi-parameters in every Point of that Semi-circumference will be equal to what they were in the other: That is at the Point i the Semi-parameter is $Xi = Ze = XV$, and at S it will be $SC = ZC = fq$, also at d , $dh = ZK = mn$, &c. hence the mixt lin'd Figure $ZSNMC$ is equal to the other mixt lin'd Figure $ZqVMNf$, and each to the Triangle ZNM .

From the same Equality of the Semi-parameters gy and pr , &c. taken at any Height and their Equality to the right Line ab , &c. it follows that the right Lines $br = by$. $ln = lb$. $fq = fC$. $OV = OX$, &c. as also that Lines, or double Ordinates, yr . hu . Cq . &c. are still equal to the corresponding
Chords

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Chords gp . dm . Sf . &c. of the Circle. From the first of these Equations it follows that the Curve in which are all the outward or Right-hand Extremities of the Parameters, &c. is the Periphery of an Ellipsis, and by the 2d. that Ellipsis is equal to a Circle whose Radius is the Impetus of the Engine.

Q. E. D.

Beside the Equations before mention'd, I see no End to what might be found in this Figure, as that the Quadrantal Segment Zfp of the Circle is equal to the Elliptical Segment ZCy , (or qMV) and that each Part of those Segments cut off or lying between the same Parallels yp . bm . &c. are still equal. Also if right Lines are drawn from Z to any two corresponding Points in the Peripheries of the Circle and Ellipsis, as Zp . Zr . or Zm . Zn . the Segments cut off by those right Lines will be equal; and if right Lines were drawn from Z and C to q and M , the Parallelogram $ZqMC$ wou'd be the greatest that cou'd be inscrib'd in the Ellipsis, and equal to a Square inscrib'd in the Circle; Each of the Segments cut off by the Sides of the Parallelogram wou'd be equal to the Quadrantal Segment of the Circle. As also that the Space $ZBqr$ is equal to the Space $fMNX$, and $fMX = CXN$, &c.

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IV.

If right Lines are suppos'd to be drawn from the Place of the Engine *A* (*Fig. 13.*) through the Focus Points of every Parabola that can be describ'd by the same Impetus *AZ*, and terminated by the Curve of each respective Parabola, as at *X*; those Points of Termination will all be in the Curve of a Parabola, whose Focus is *A*, Vertex *Z*, and principal Parameter four Times the Impetus *AZ*.

By *Cor. 4. Theo. 10.* all the parabola's that can be describ'd by the Impetus *AZ* have their Focus Points in the Circumference of the Circle *AZFN*, &c.

Let $Af = x$, f being taken any where in that Circumference, and $fX = XR$ (*Cor. 2. Theo. 8*) $= y$; then will $AX = x + y$, and $YX = x - y$. but (by 47. 1. *El.*) $AX^2 - YX^2 = AY^2 = 2X^2$; (*per Figure*) that is, in Symbols, $4xy = 2X^2$; but $2Z = XR = y$, and $4x = 4AZ$, therefore $4AZ \times Z = 2X^2$, whence, and from *Theo. 7.* the Point *X* is in the Curve of a Parabola whose Focus is *A*, Vertex *Z*, and principal Parameter four Times the Impetus *AZ*.

F I N I S.

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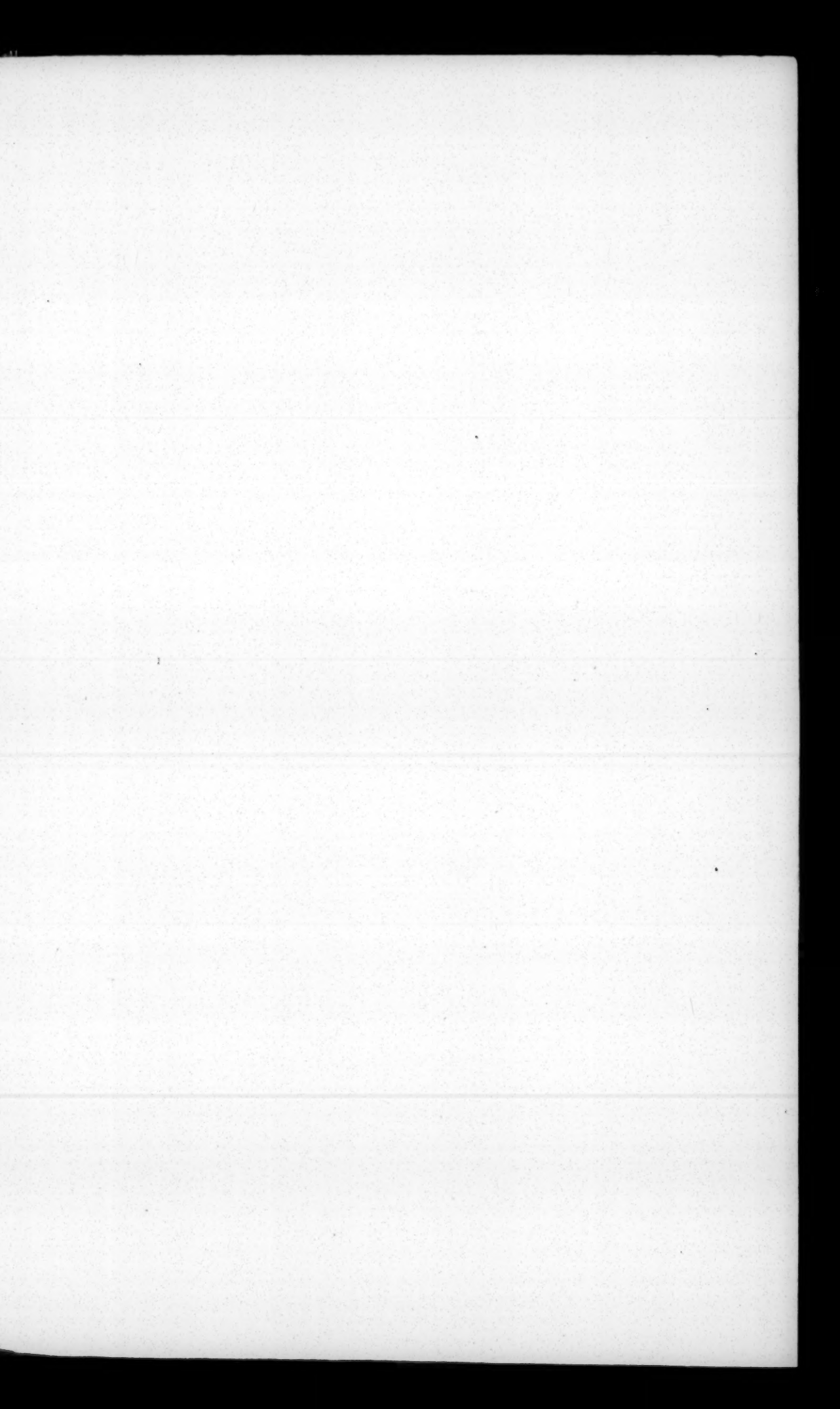




Fig. 1.

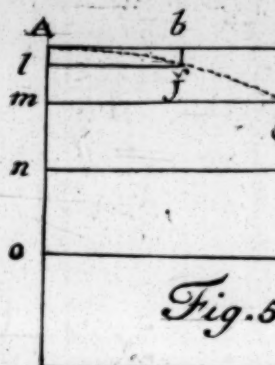


Fig. 5.



Fig. 2.

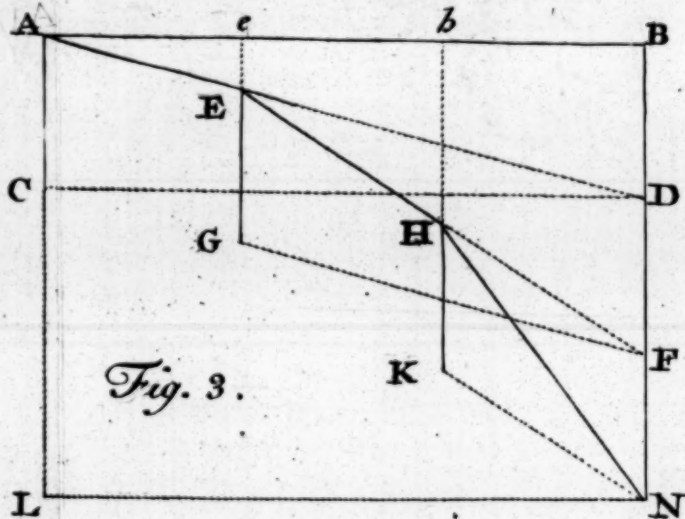


Fig. 3.

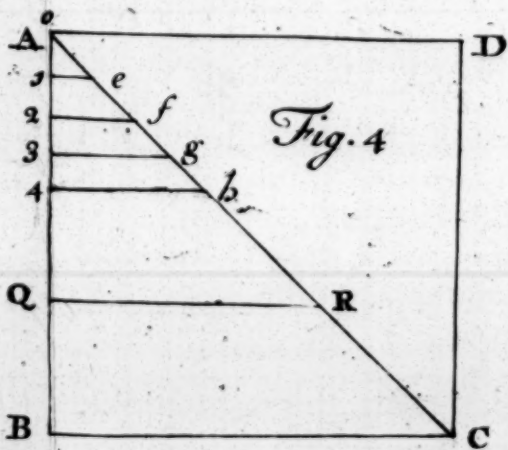
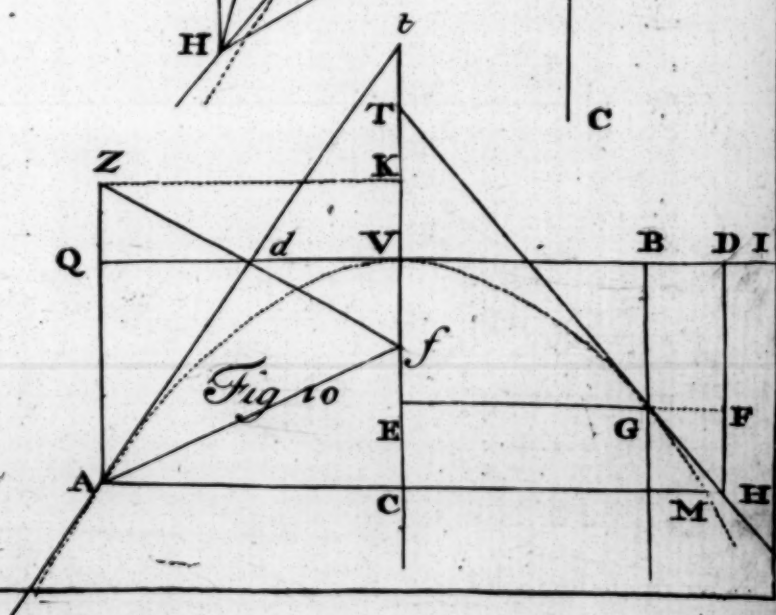
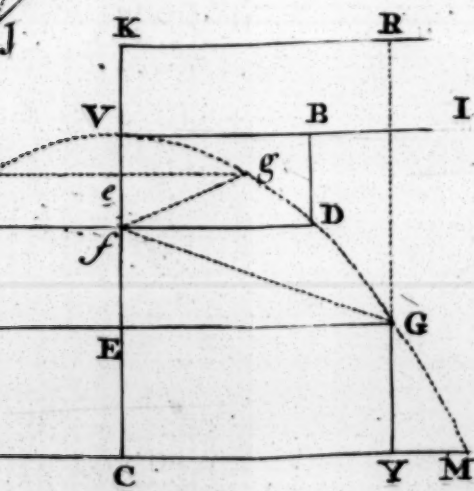
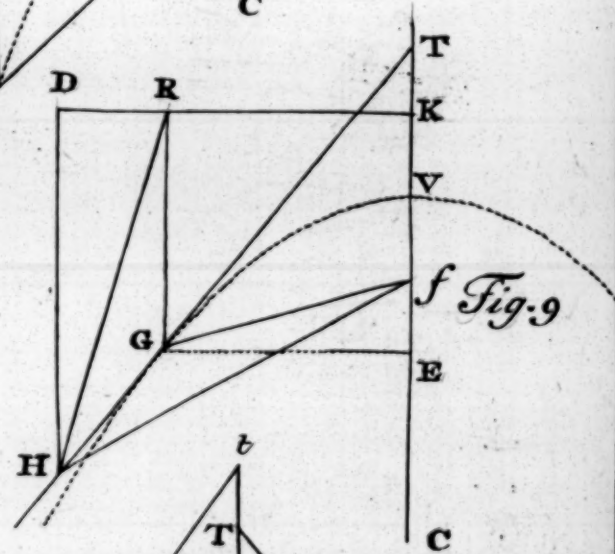
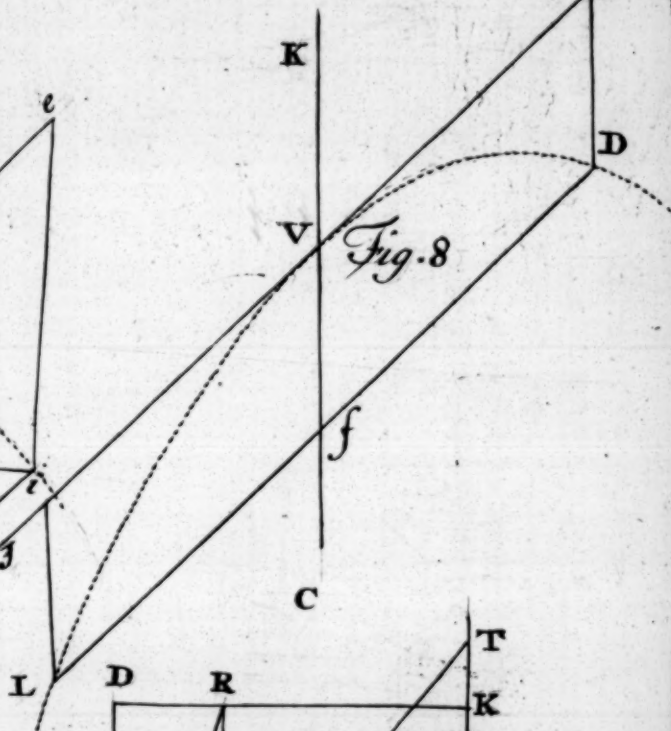
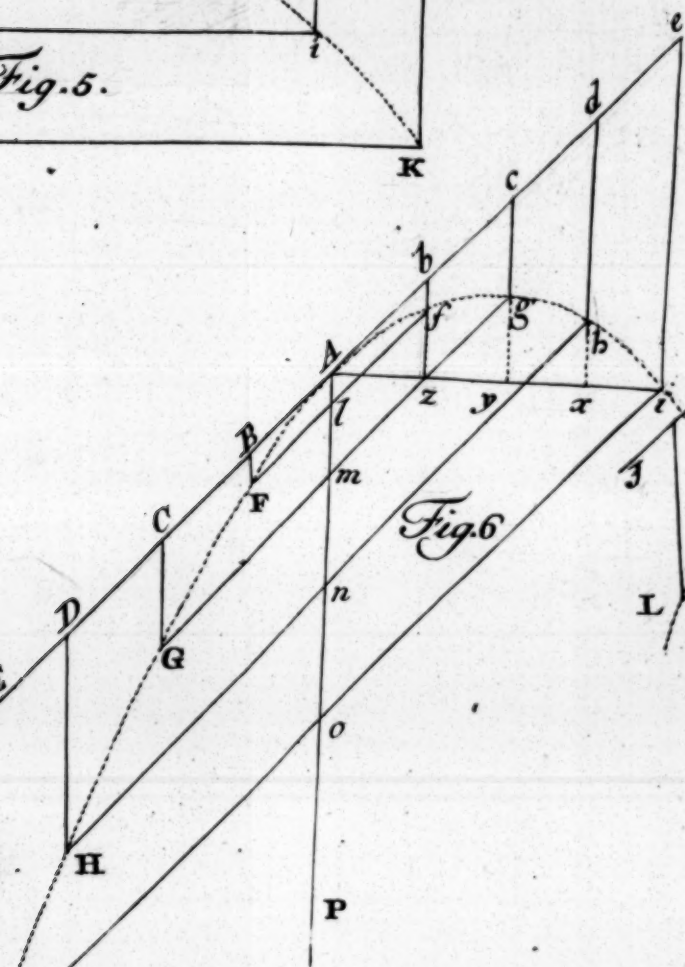
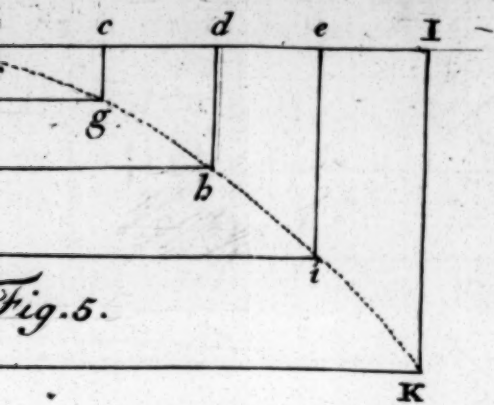


Fig. 4



Fig. 7



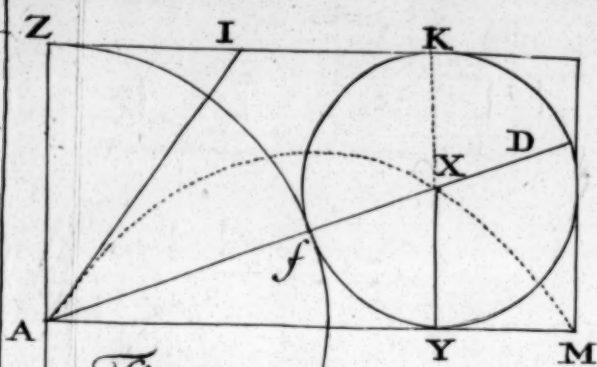


Fig 11.

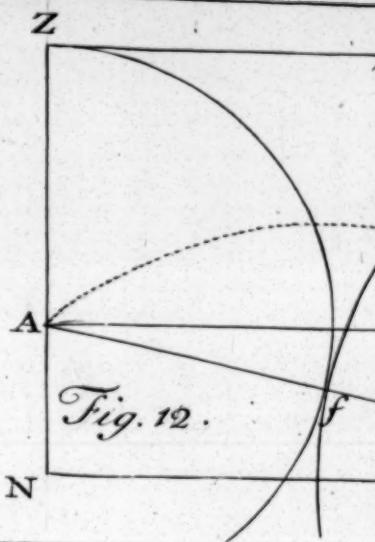


Fig. 12.

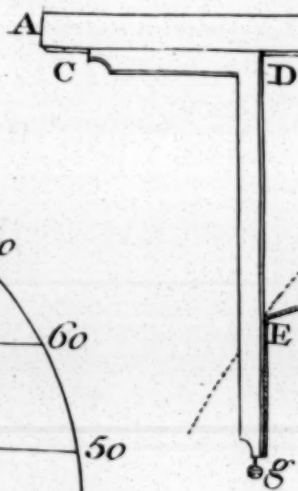


Fig 15.

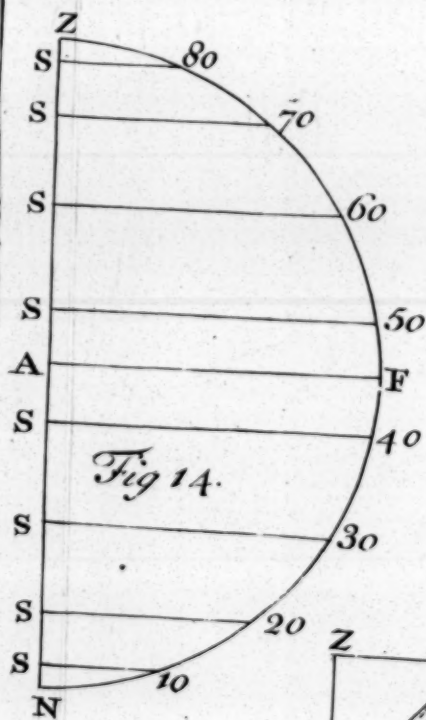


Fig 14.

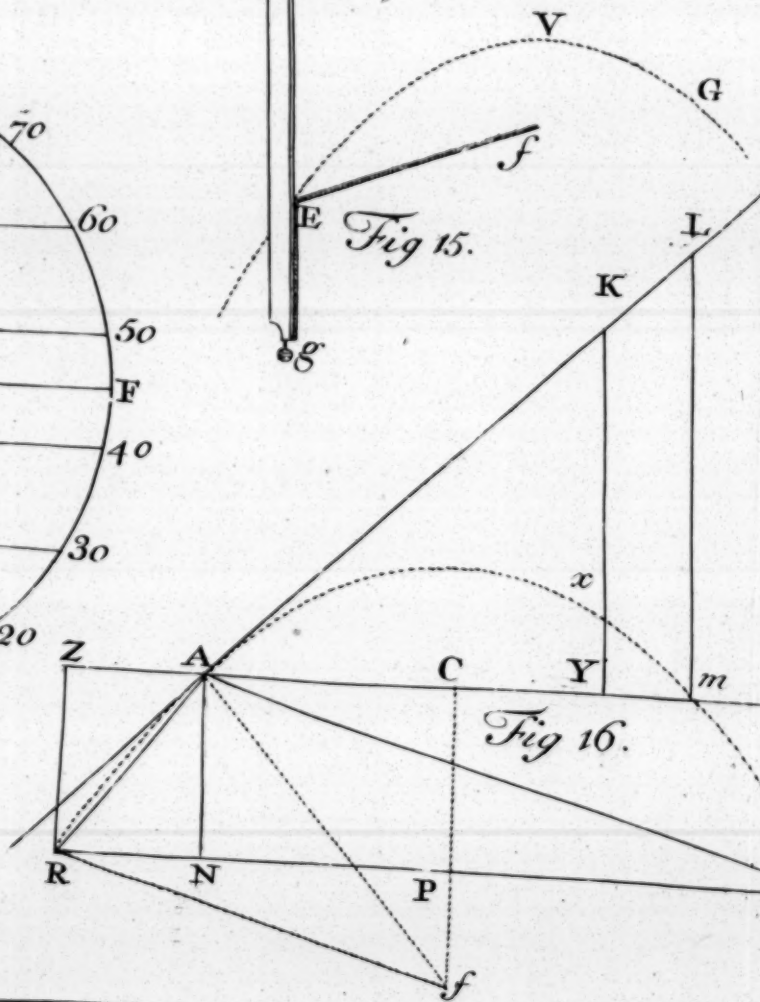


Fig 16.

